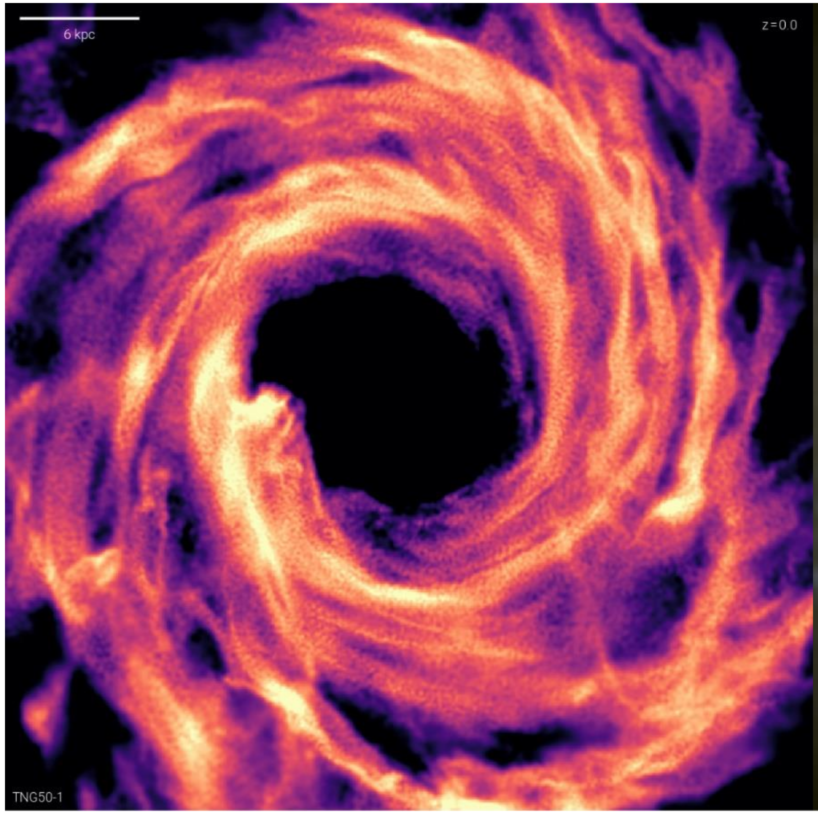
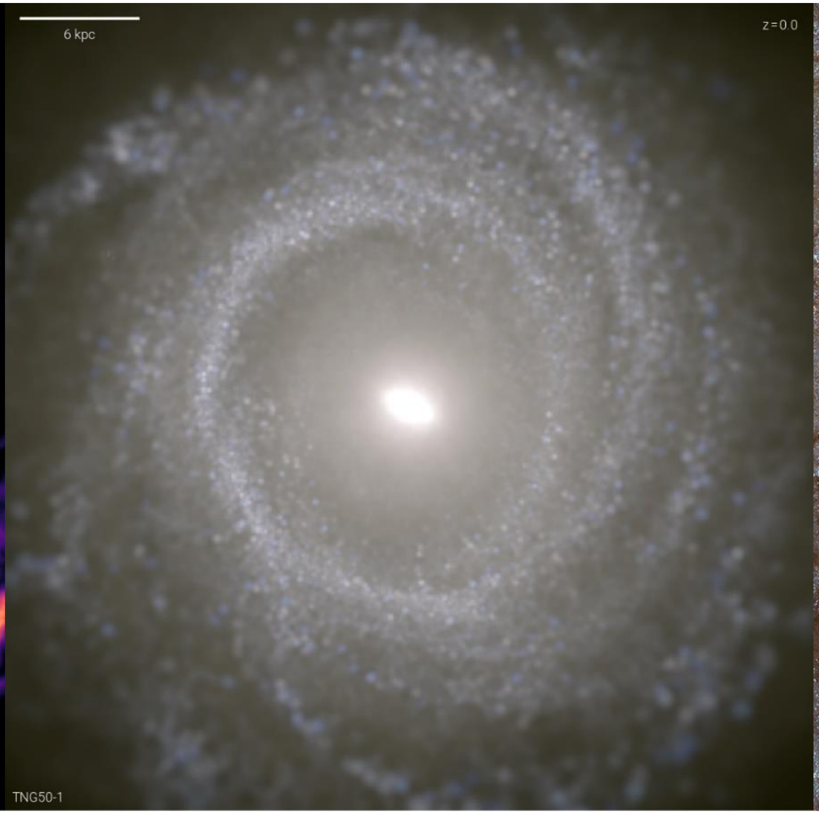
MOAA Guest Lecture

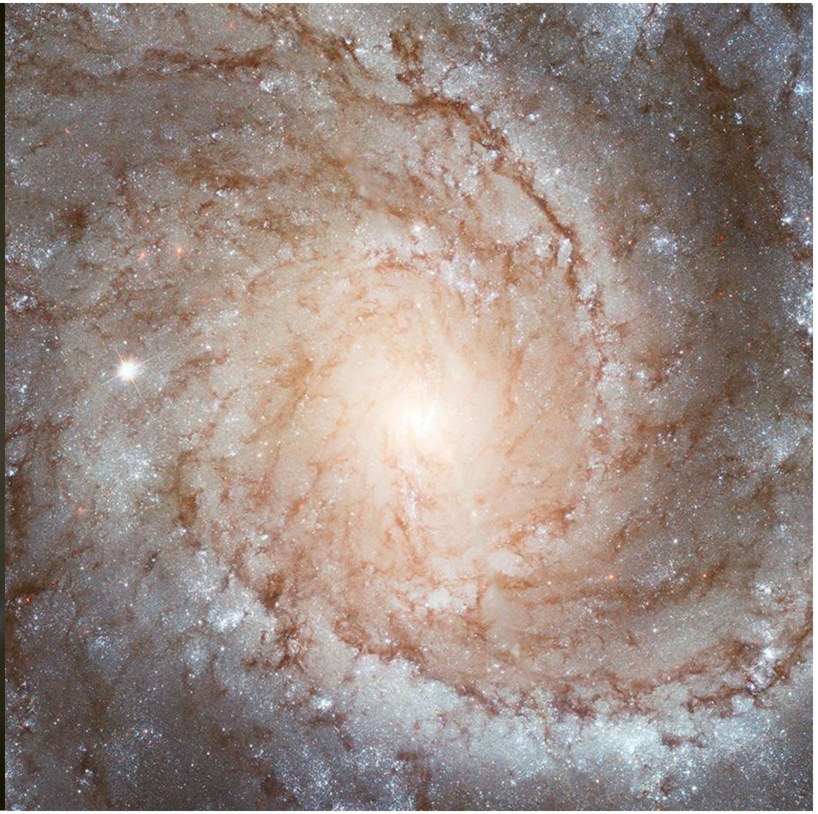
Radiative Processes

2025.03.22 | Yen-Hsing Lin (UCSD)

Why do we care about radiative processes?



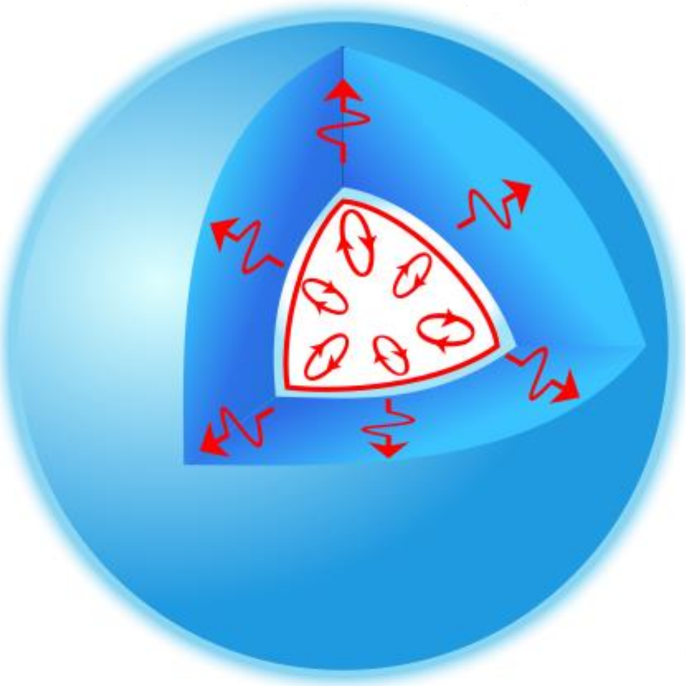




EM waves are our primary tool for understanding the universe.
We need to know how to convert photons into meaningful physical properties.

illustrisTNG | NASA, ESA, K. Kuntz, F. Bresolin, J. Trauger, J. Mould, Y.-H. Chu, CFHT/J.-C. Cuillandre/Coelum, and G. Jacoby, B. Bohannan, and M. Hanna/NOAO/AURA/NSF

Why do we care about radiative processes?



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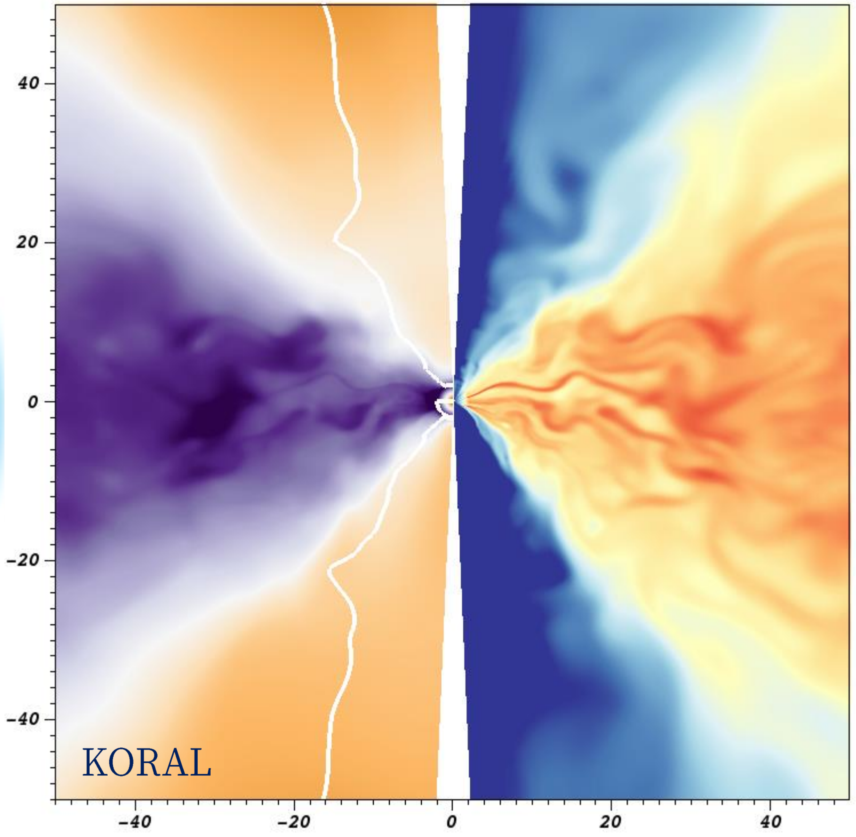
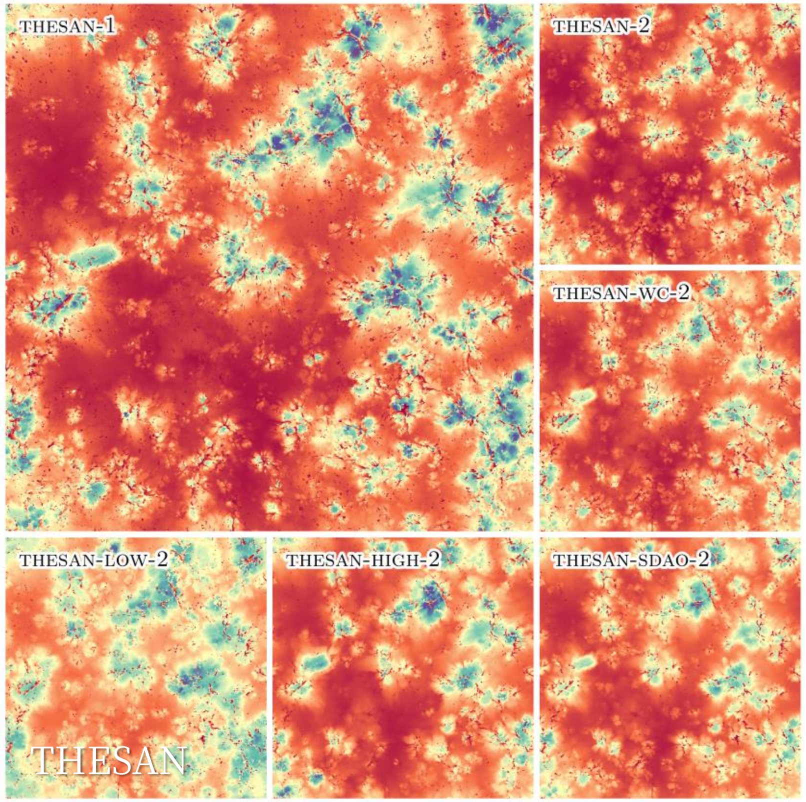




Radiation plays important roles in multiple astrophysical systems.

What does **bright** actually mean?

In physical sciences [edit]

Physics [edit]

- Intensity (physics), power per unit area (W/m^2)
- Field strength of electric, magnetic, or electromagnetic fields (V/m, T, etc.)
- Intensity (heat transfer), radiant heat flux per unit area per unit solid angle ($W{\cdot}m^{-2}{\cdot}sr^{-1}$)
- Electric current, whose value is sometimes called *current intensity* in older books

Optics [edit]

- Radiant intensity, power per unit solid angle (W/sr)
- Luminous intensity, luminous flux per unit solid angle (lm/sr or cd)
- Irradiance, power per unit area (W/m^2)

Astronomy [edit]

- Radiance, power per unit solid angle per unit projected source area ($W{\cdot}sr^{-1}{\cdot}m^{-2}$)

Seismology [edit]

- Mercalli intensity scale, a measure of earthquake impact
- Japan Meteorological Agency seismic intensity scale, a measure of earthquake impact
- Peak ground acceleration, a measure of earthquake acceleration (g or m/s^2)

Acoustics [edit]

- Sound intensity, sound power per unit area

SI photometry quantities

V · T · E

Quantity		Unit		Dimension [nb 1]	Notes
Name	Symbol [nb 2]	Name	Symbol		
Luminous energy	Q_v [nb 3]	lumen second	lm⋅s	T⋅J	The lumen second is sometimes called the *talbot*.
Luminous flux, luminous power	Φ_v [nb 3]	lumen (= candela steradian)	lm (= cd⋅sr)	J	Luminous energy per unit time
Luminous intensity	I_v	candela (= lumen per steradian)	cd (= lm/sr)	J	Luminous flux per unit solid angle
Luminance	L_v	candela per square metre	cd/m^2 (= $lm/(sr{\cdot}m^2)$)	$L^{-2}{\cdot}J$	Luminous flux per unit solid angle per unit *projected* source area. The candela per square metre is sometimes called the *nit*.
Illuminance	E_v	lux (= lumen per square metre)	lx (= lm/m^2)	$L^{-2}{\cdot}J$	Luminous flux *incident* on a surface
Luminous exitance, luminous emittance	M_v	lumen per square metre	lm/m^2	$L^{-2}{\cdot}J$	Luminous flux *emitted* from a surface
Luminous exposure	H_v	lux second	lx⋅s	$L^{-2}{\cdot}T{\cdot}J$	Time-integrated illuminance
Luminous energy density	ω_v	lumen second per cubic metre	$lm{\cdot}s/m^3$	$L^{-3}{\cdot}T{\cdot}J$	
Luminous efficacy (of radiation)	K	lumen per watt	lm/W	$M^{-1}{\cdot}L^{-2}{\cdot}T^3{\cdot}J$	Ratio of luminous flux to radiant flux
Luminous efficacy (of a source)	η [nb 3]	lumen per watt	lm/W	$M^{-1}{\cdot}L^{-2}{\cdot}T^3{\cdot}J$	Ratio of luminous flux to power consumption

How do we make sense of all these quantities?

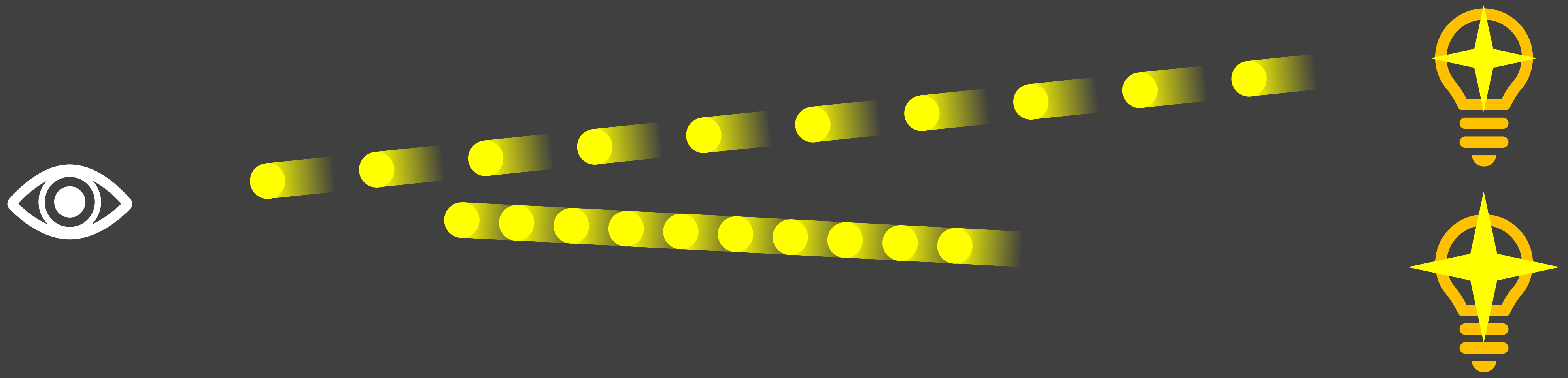
How should we define **bright**?

$$\text{Brightness} \propto \Delta E \quad ?$$

Maybe **bright** means we receive more energy from the light source?

How should we define **bright**?

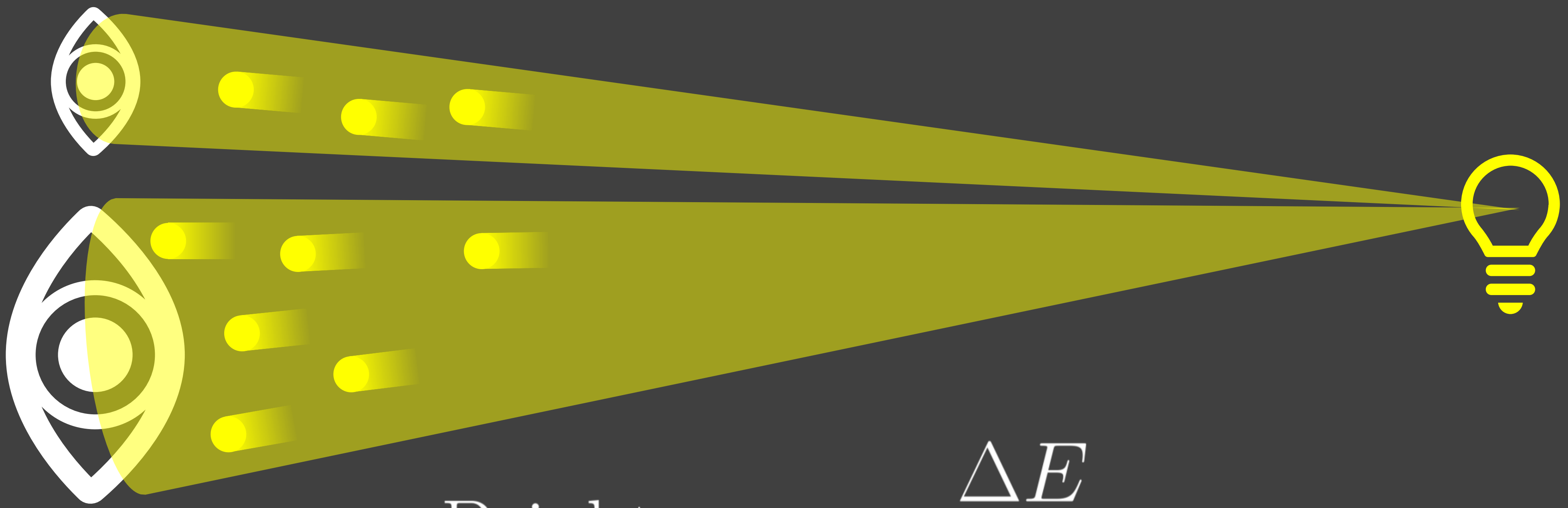
Things appear brighter if you receive
the same amount of energy in a shorter time.



$$\text{Brightness} \propto \frac{\Delta E}{\Delta t}$$

How should we define **bright?**

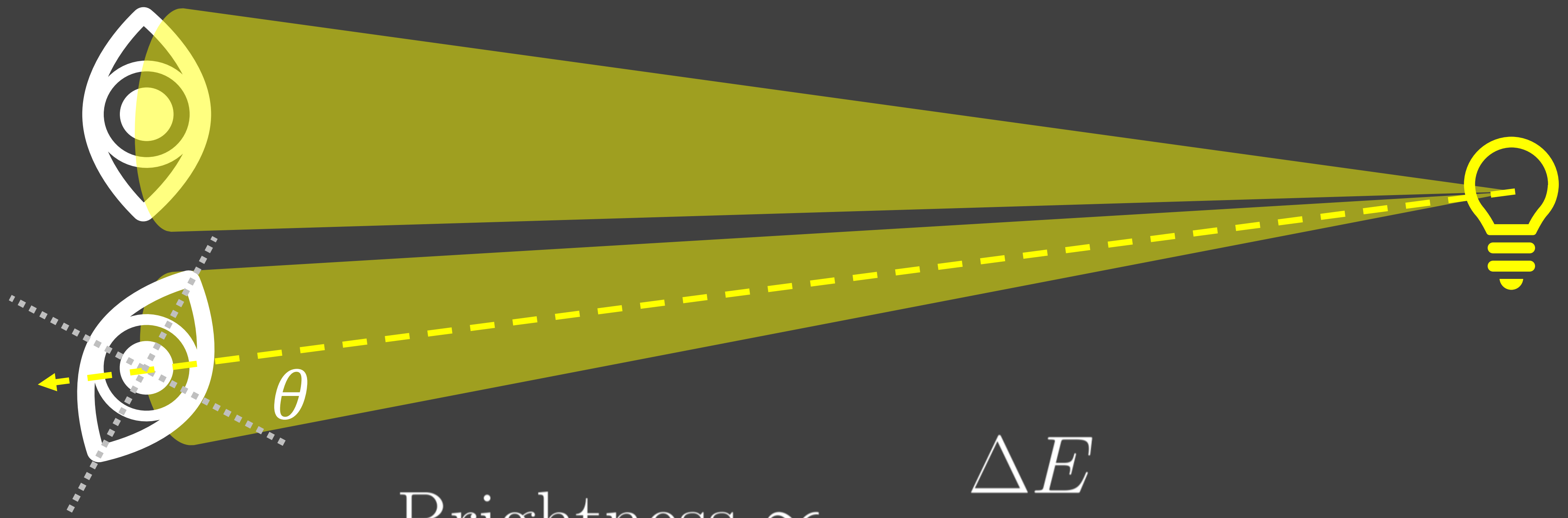
A larger collecting area collects more photons from the same light source.



$$\text{Brightness} \propto \frac{\Delta E}{\Delta A \Delta t}$$

How should we define **bright?**

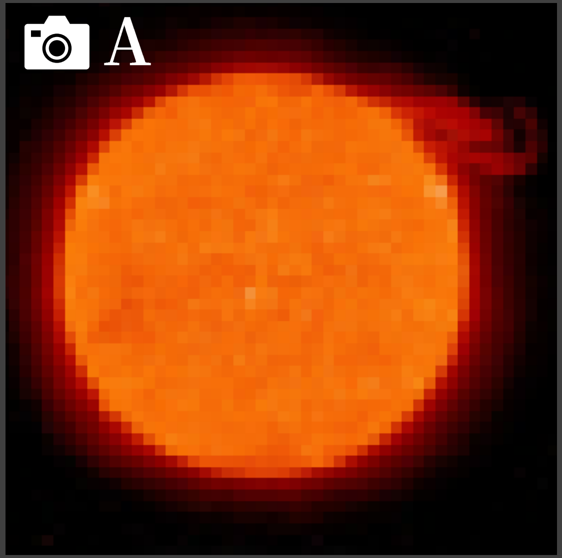
Tilted collecting area is effectively smaller.

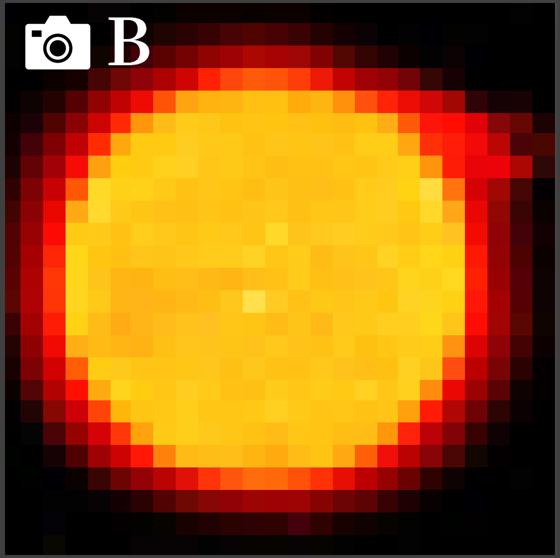


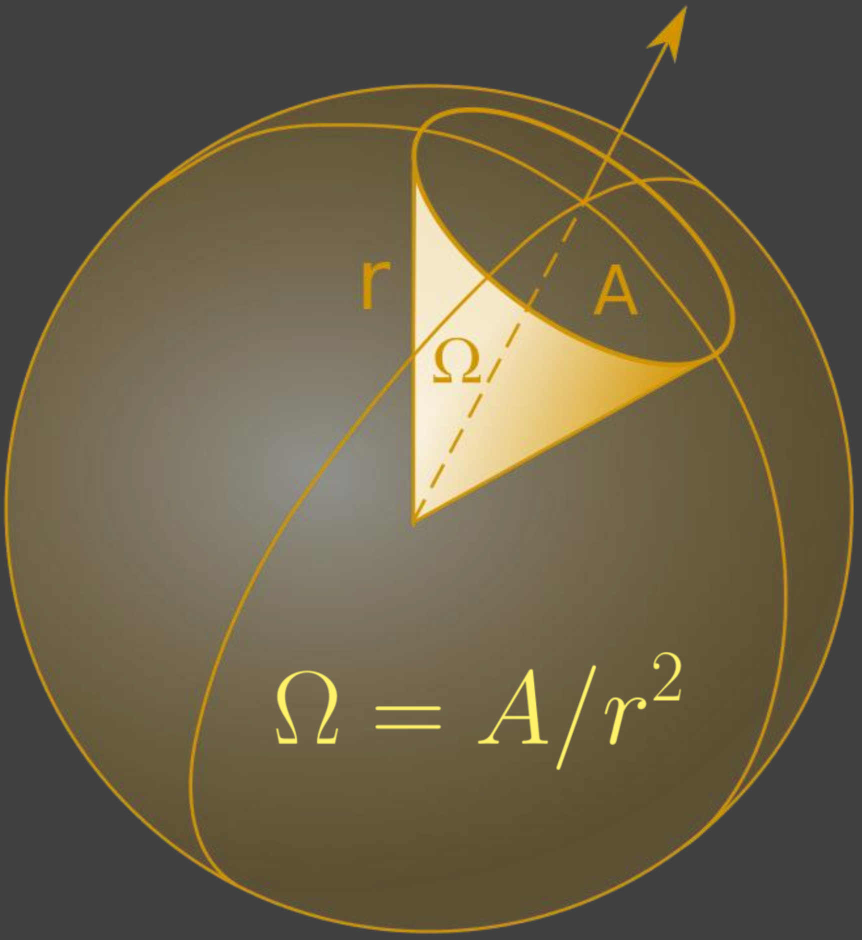
$$\text{Brightness} \propto \frac{\Delta E}{\cos\theta \Delta A \Delta t}$$

How should we define **bright?**

Consider taking the picture of the sun with 2 different camera.



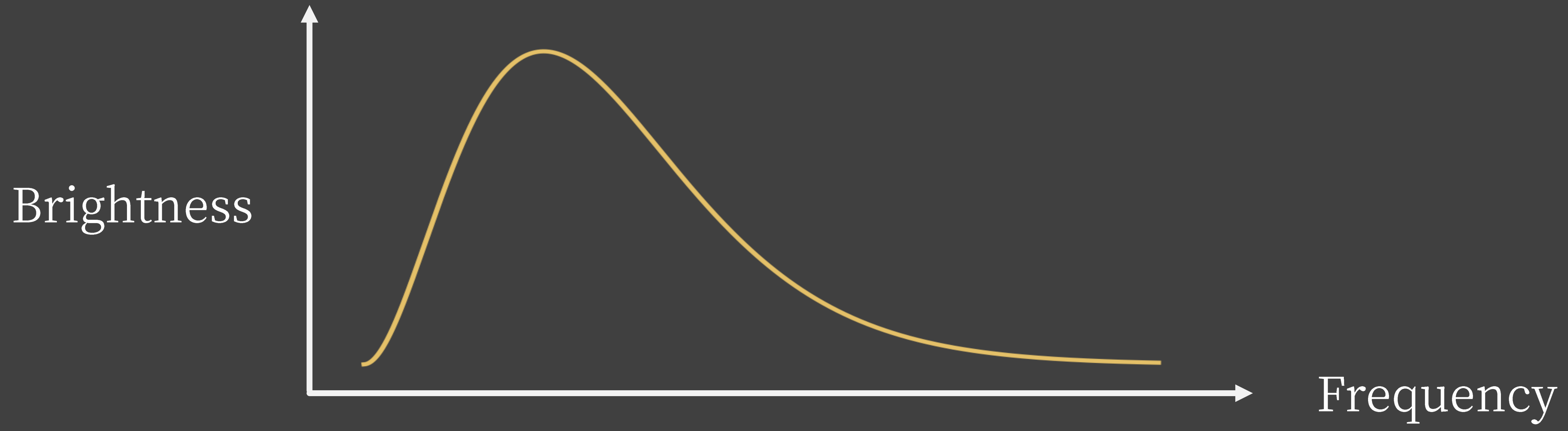




$$\text{Brightness} \propto \frac{\Delta E}{\cos\theta \Delta A \Delta t \Delta \Omega}$$

How should we define **bright?**

Finally, brightness should be a function of frequency/wavelength.



$$\text{Brightness} \propto \frac{\Delta E}{\cos\theta \Delta A \Delta t \Delta \Omega \Delta \nu}$$

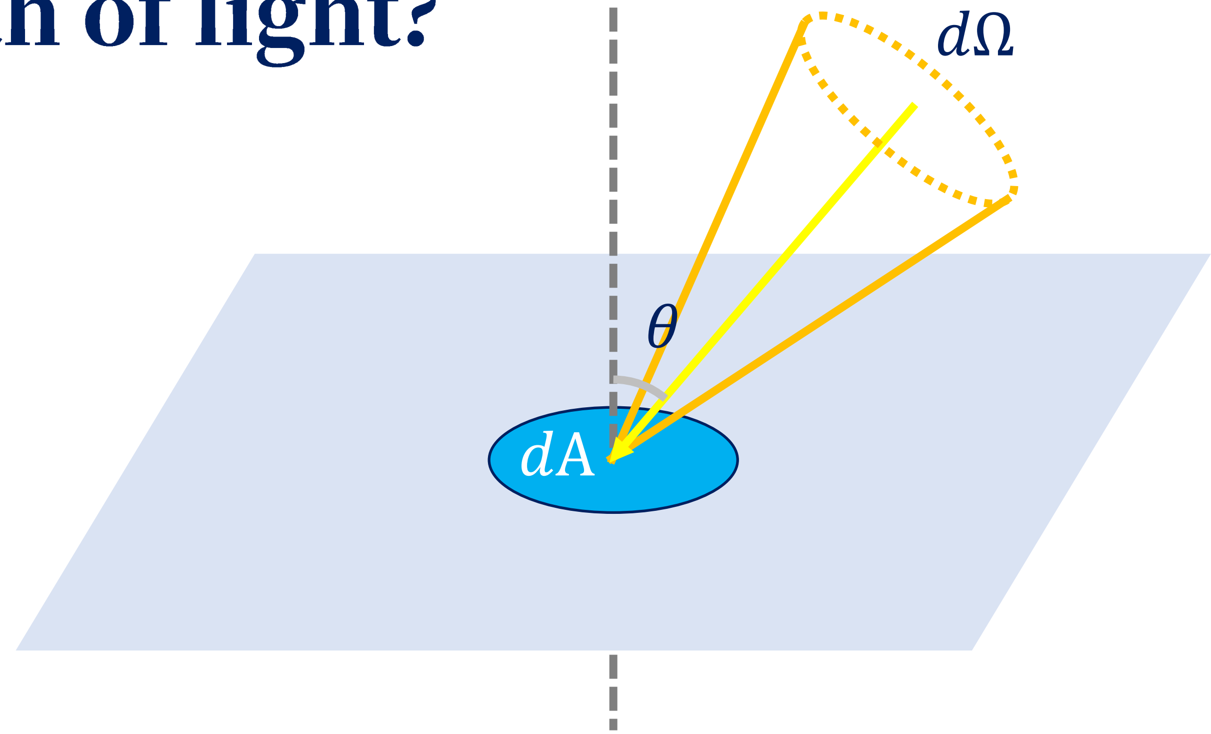
How to define the strength of light?

The energy we receive is affected by

- **Integration time**
- **Direction**
- **Collecting area**
- **Solid angle**
- **Frequency**

From this, we define:

Specific Intensity (I_ν)



$$I_\nu \equiv \frac{dE}{\cos\theta dAdtd\Omega d\nu}$$

Other quantities

$$\frac{dE}{\cos\theta\; dt\, dA\, d\Omega\, d\nu} = I_\nu$$

1. Energy Received (E). Unit: [J].
2. Power (P). Unit: [J s^{-1}].
3. Flux (F). Unit: [J m^{-2} s^{-1}].
4. Total Intensity (I). Unit: [J m^{-2} sr^{-1} s^{-1}]. Also called **Surface Brightness**.
5. Specific Intensity (I_ν). Unit: [J m^{-2} sr^{-1} s^{-1} Hz^{-1}].

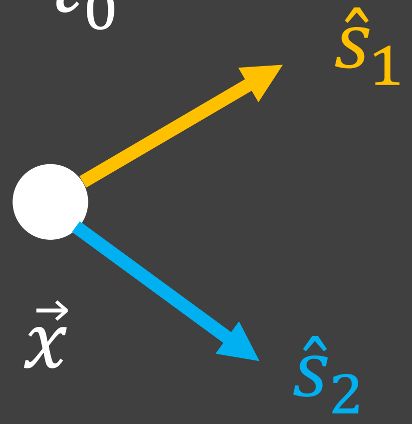
The **names** are not important. What you should care about is the **units.**

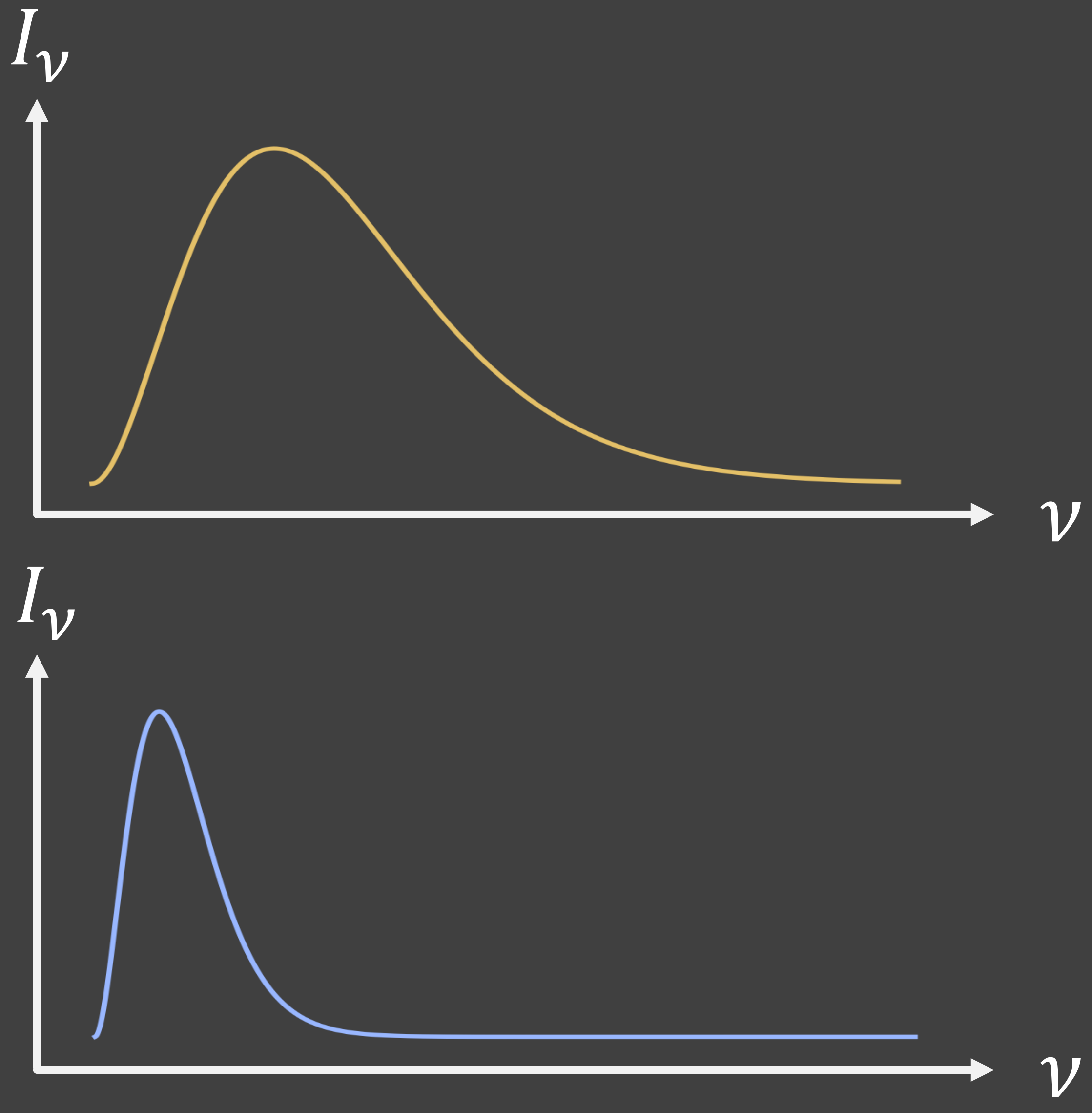
I_ν is convenient because it is an **intrinsic** property of the source.

Specific intensity is a function of position, direction, frequency, and time.

$$I_\nu = I_\nu(\vec{x}, \hat{s}, \nu, t)$$

$t = t_0$





Key property: specific intensity does not decay with distance!!!

With no absorption/emission, specific intensity remains constant

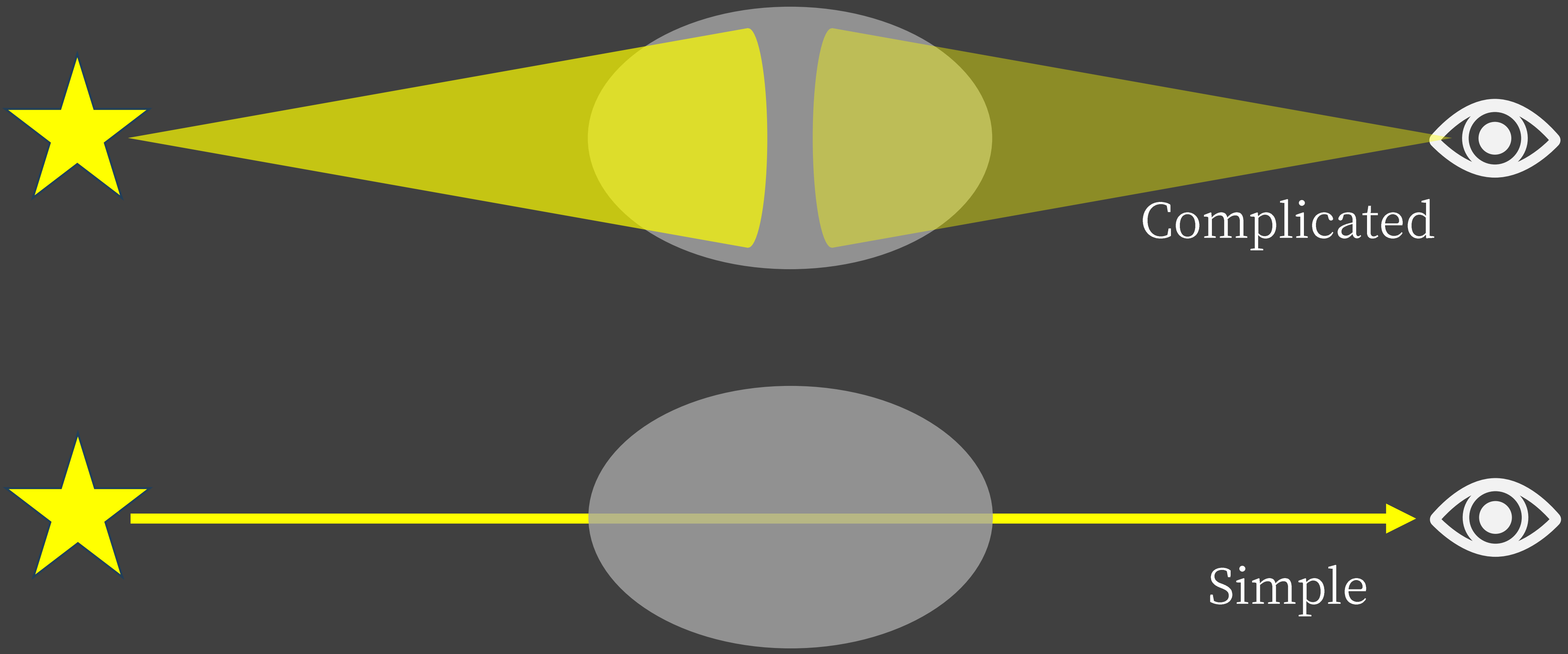
Flux decreases with $1/r^2$,

but the solid angle of the source also decreases with $1/r^2$.

$$I_\nu = \frac{F_\nu}{\Omega} = \frac{F_0 (r/r_0)^{-2}}{\Omega_0 (r/r_0)^{-2}} = \text{const.}$$

But how is that useful?

That simplifies the problem into 1D.



Summary 01

How do we define brightness?

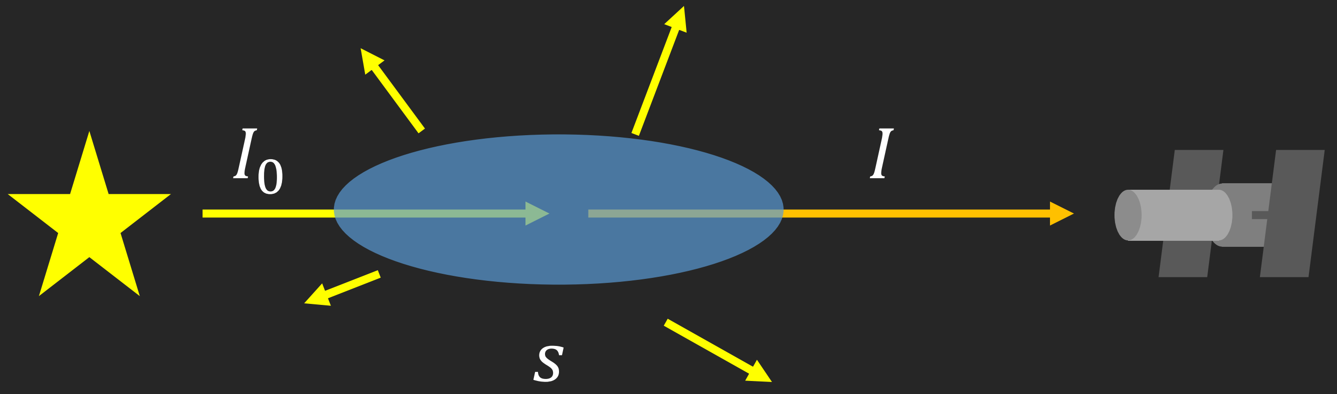
$$I_\nu \equiv \frac{dE}{\cos\theta dA dt d\Omega d\nu}$$

1. Specific intensity is commonly used in astronomy.
2. Specific intensity is defined as the **energy** received per unit **time**, **frequency**, **area**, and **solid angle**.
3. Specific intensity **does not** decay with distance!
4. Why should you know specific intensity?
   - ➢ You better be able to understand what people are saying.
   - ➢ You want to find invariant-like quantity in a complicated system, so that the problems are simplified, and you get physical intuition.

Change of intensity: Radiative transfer

What changes the specific intensity?

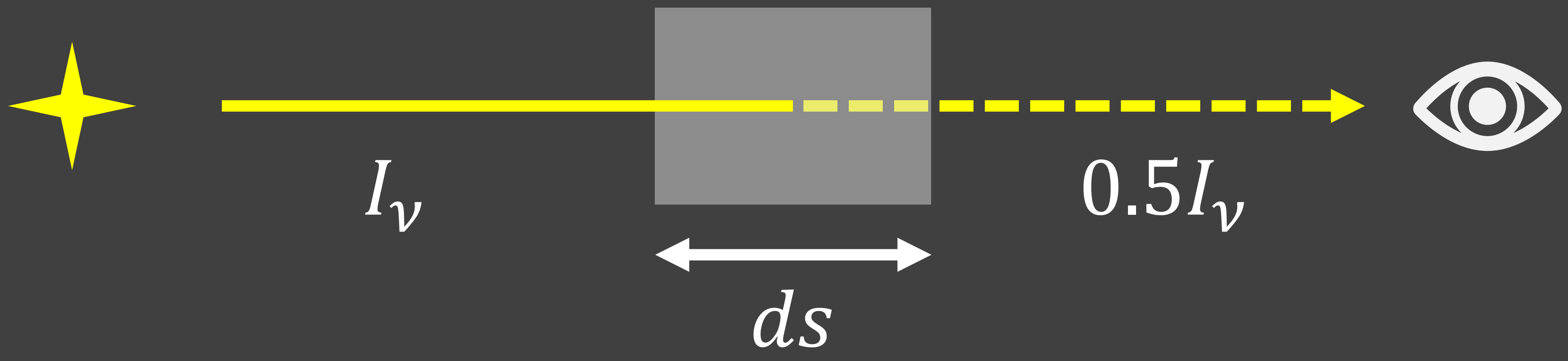
Emission, **Absorption** and **Scattering**.



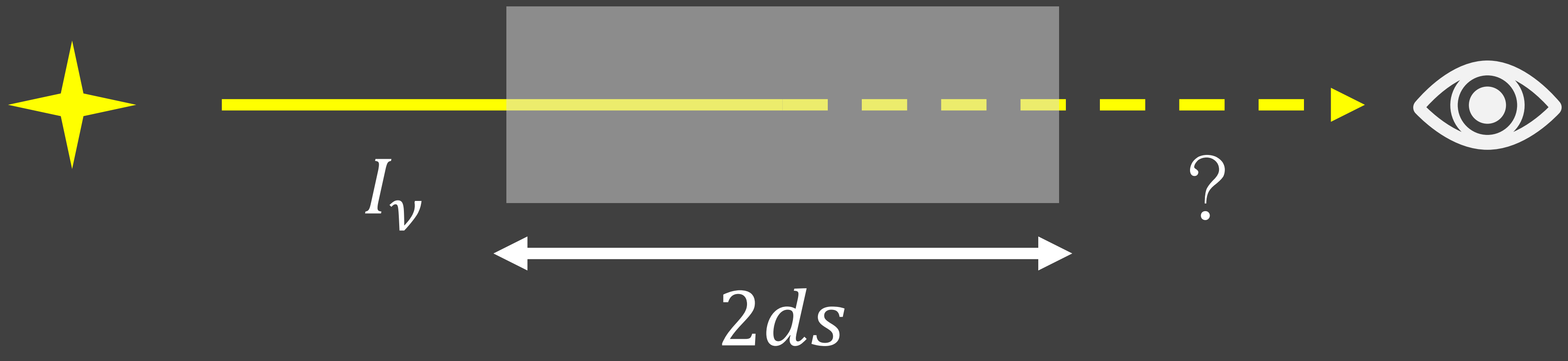
$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

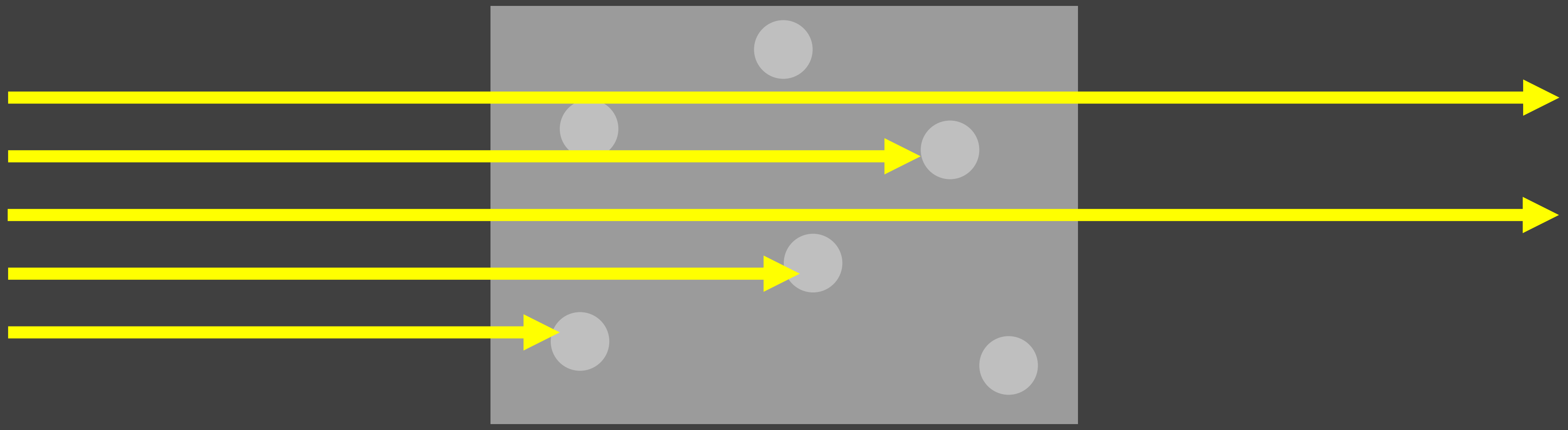
Case: pure absorption

Consider a gas cloud with length ds absorbed 50% of the incident light.

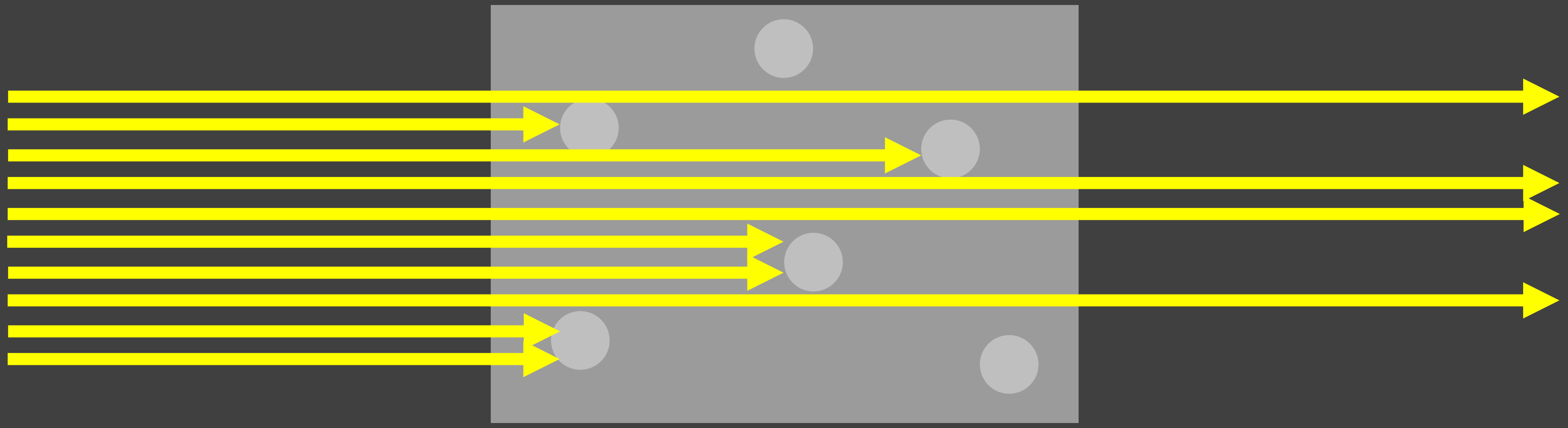


Now if the cloud is 2 times longer, what would the final intensity be?





The fraction of light being absorbed is fixed.



So to put our intuition into the language of physics,
we can write:

$$dI_\nu = -\alpha_\nu I_\nu ds$$

Where α_ν is called the **absorption coefficient**.
That describe how **opaque** the gas cloud / medium is.

We can further express the absorption coefficient as:

$$\alpha_\nu = \kappa_\nu \rho = \sigma_\nu n$$

Opacity

[cm^2 g^{-1}]

Density

[g cm^{-3}]

Cross section

[cm^2]

Number density

[cm^{-3}]

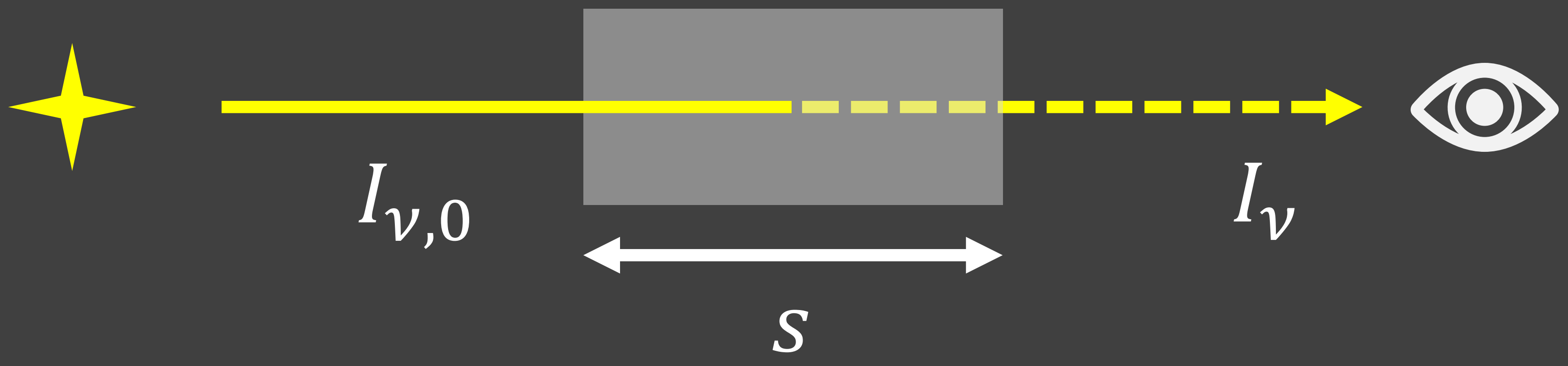
In astrophysics, people usually use opacity.

So when there is only absorption, we know:

$$\frac{dI_\nu}{ds} = -\rho\kappa_\nu I_\nu$$

In a simple case where the cloud is uniform,

what is the solution to this ODE?

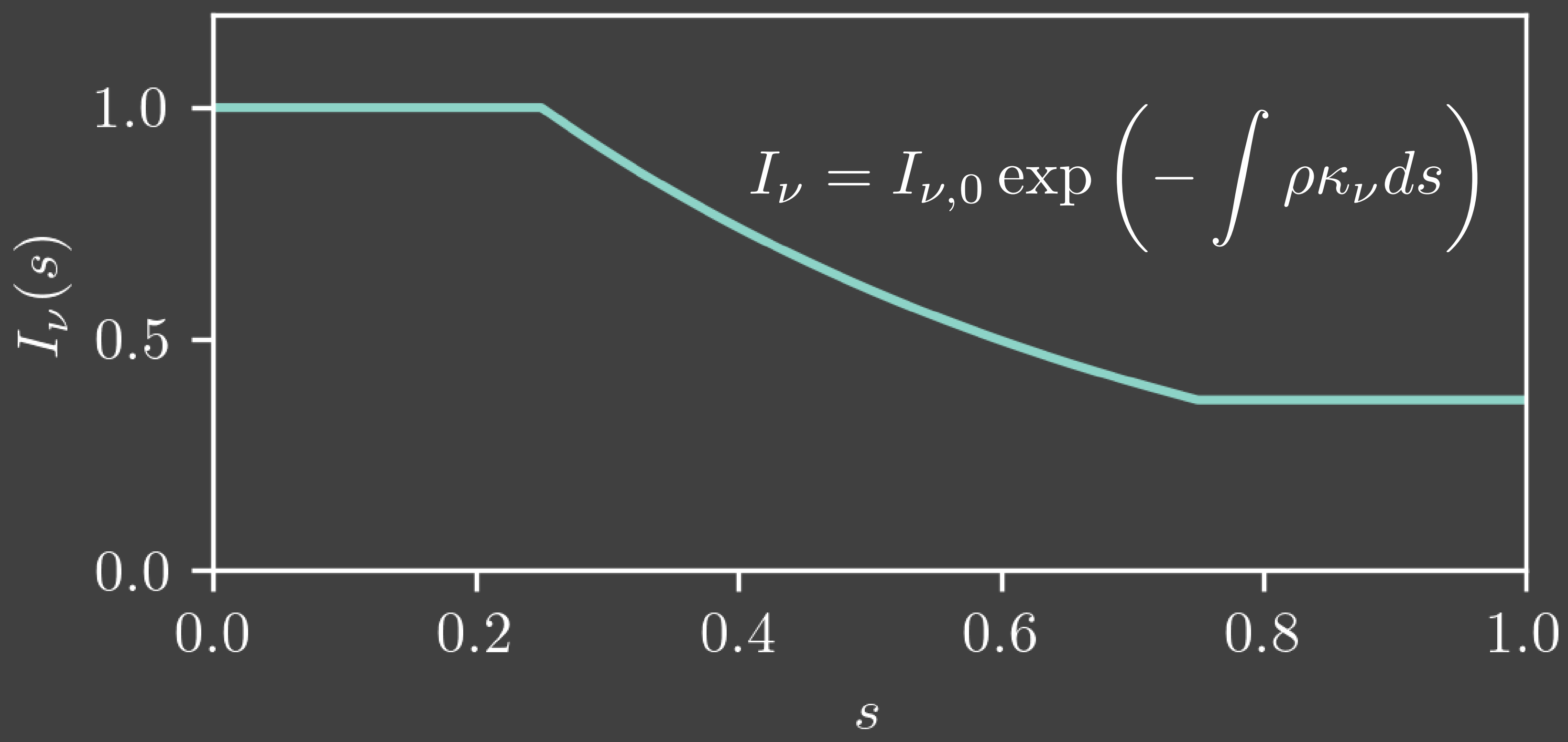


We can do:

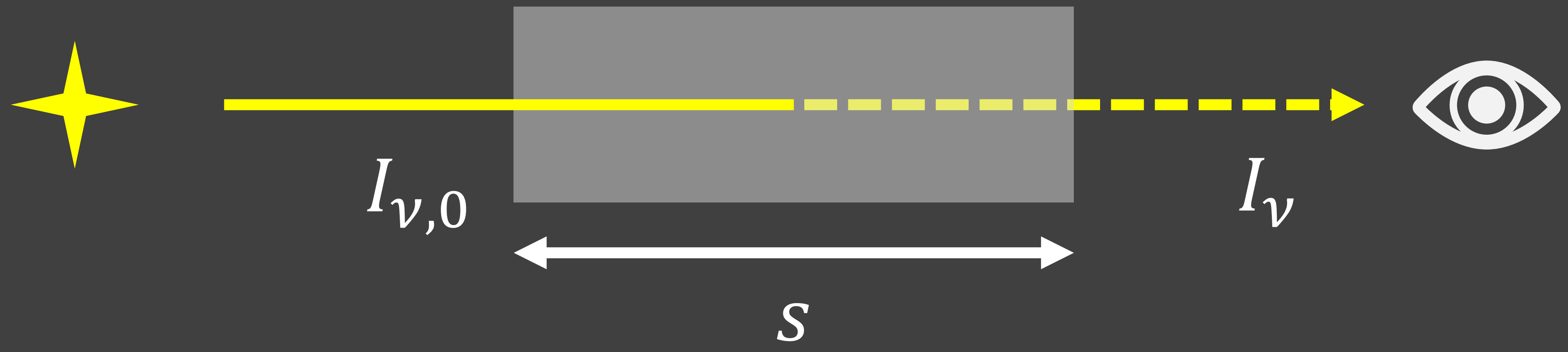
$$\frac{dI_\nu}{ds} = -\rho\kappa_\nu I_\nu$$

$$\frac{dI_\nu}{I_\nu} = -\rho\kappa_\nu ds \Rightarrow \int \frac{dI_\nu}{I_\nu} = -\int \rho\kappa_\nu ds$$

$$\ln I_\nu + C = -\int \rho\kappa_\nu ds \Rightarrow I_\nu = I_{\nu,0} \exp\left(-\int \rho\kappa_\nu ds\right)$$



$I_\nu = I_{\nu,0} \exp\left(-\int \rho\kappa_\nu ds\right)$
$I_\nu(s)$
1.0
0.5
0.0
0.0
0.2
0.4
0.6
0.8
1.0
s



$I_{\nu,0}$
I_ν
s

We therefore define the **optical depth**:

$$\tau_\nu = \int \rho \kappa_\nu ds = \ln\left(\frac{I_{\nu,0}}{I_\nu}\right)$$

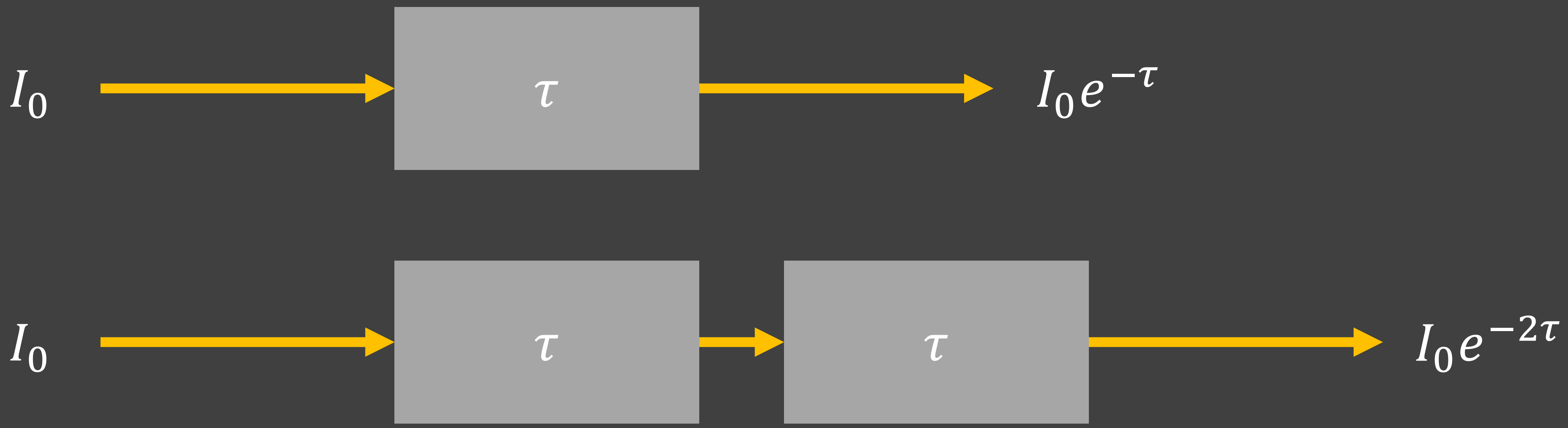
A **dimensionless** quantity that describe how much light (numbers of e-folding) is absorbed by the medium.

For example, for a gas cloud with:

$$\tau = 1 \Rightarrow I_\nu = I_{\nu,0} e^{-1} = 0.368 I_{\nu,0}$$

$$\tau = 10 \Rightarrow I_\nu = I_{\nu,0} e^{-10} = 4.540 \times 10^{-5} I_{\nu,0}$$

Optical depth is **additive.**

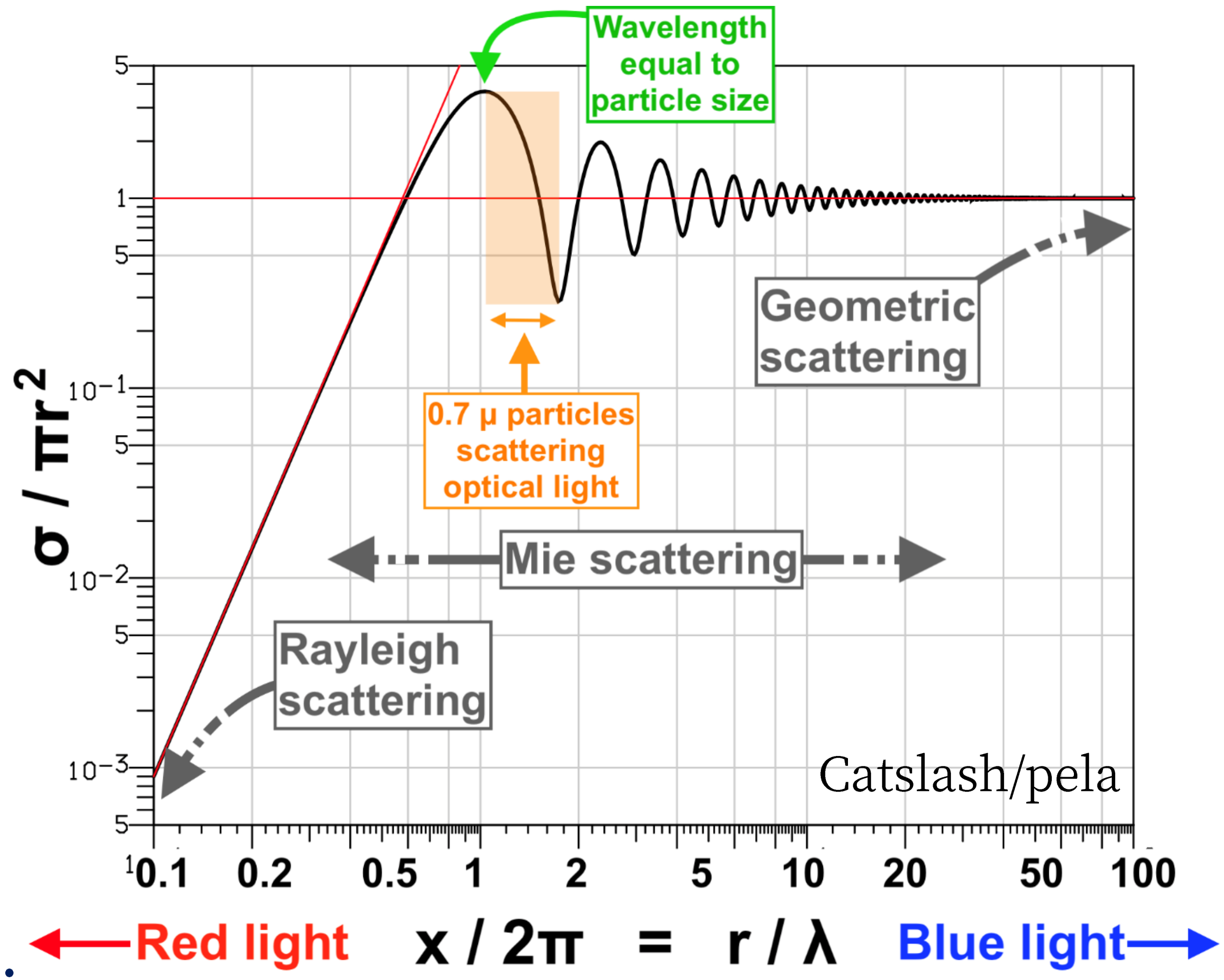


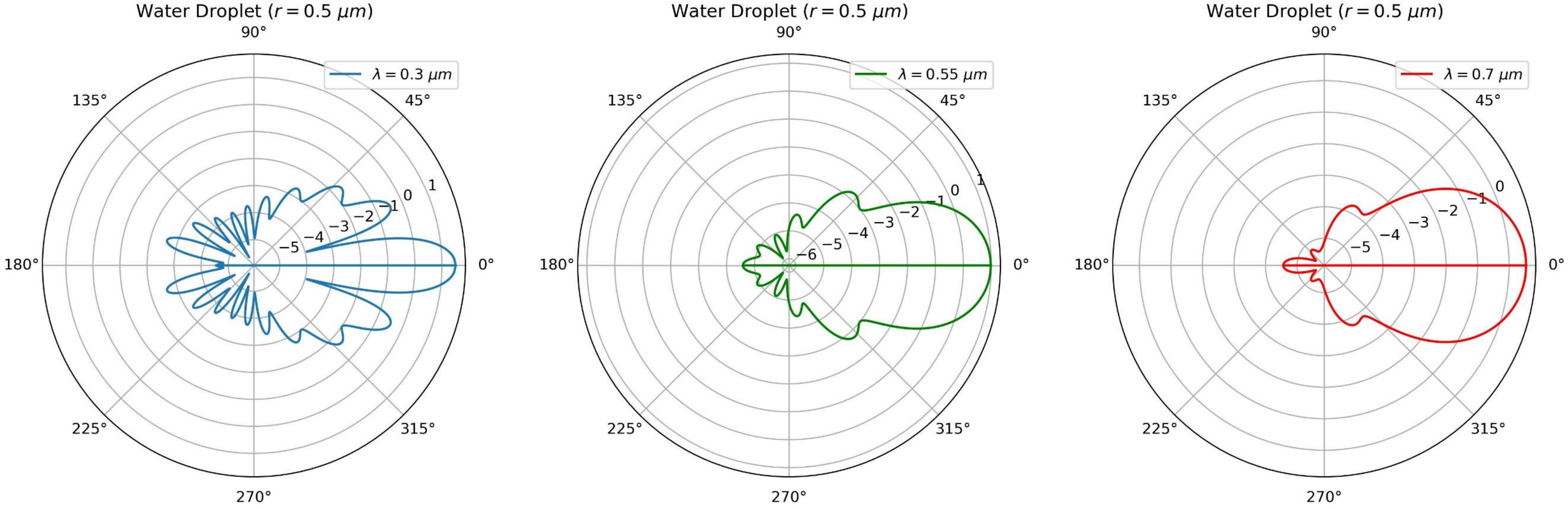
The combined absorption from the 2 clouds each with optical depth τ is just 2τ

Nightmare

Scattering

In reality, photons can be scattered back into the line of sight, which make the problem more complex. With scattering, our problem is no longer 1 dimensional, and thus we often need to consider the **complicated geometry** of our target.





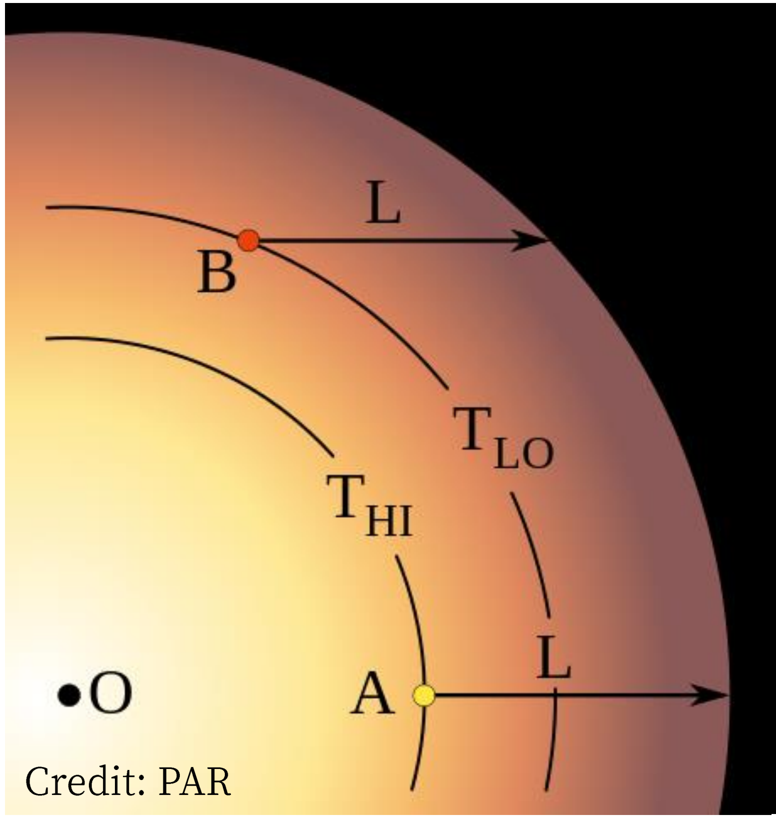
Scattering is not only wavelength dependent, but also anisotropic.

Different wavelength / grain size, creates different scattering pattern.

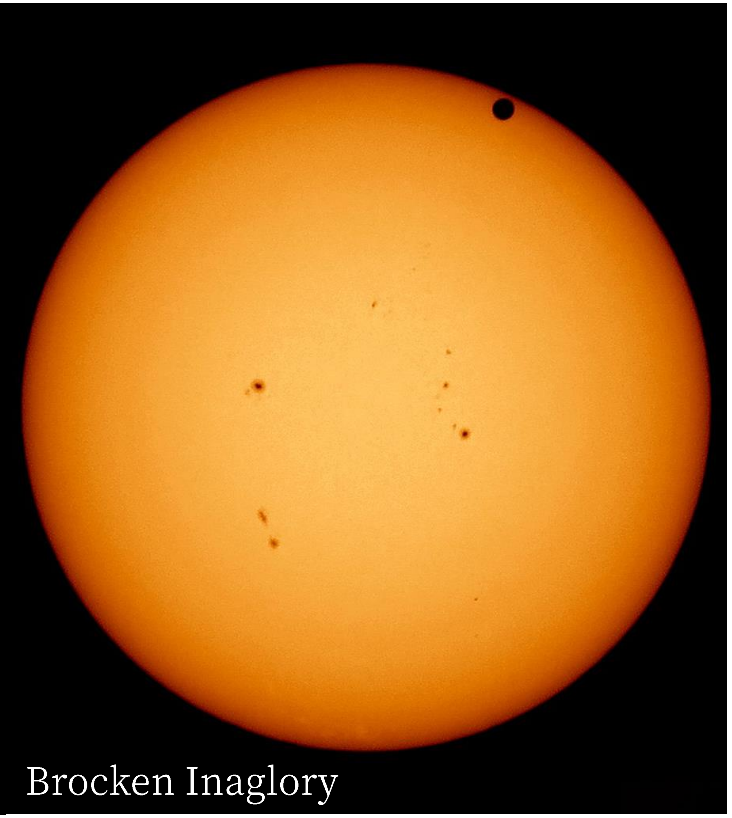
This is very hard to model analytically.

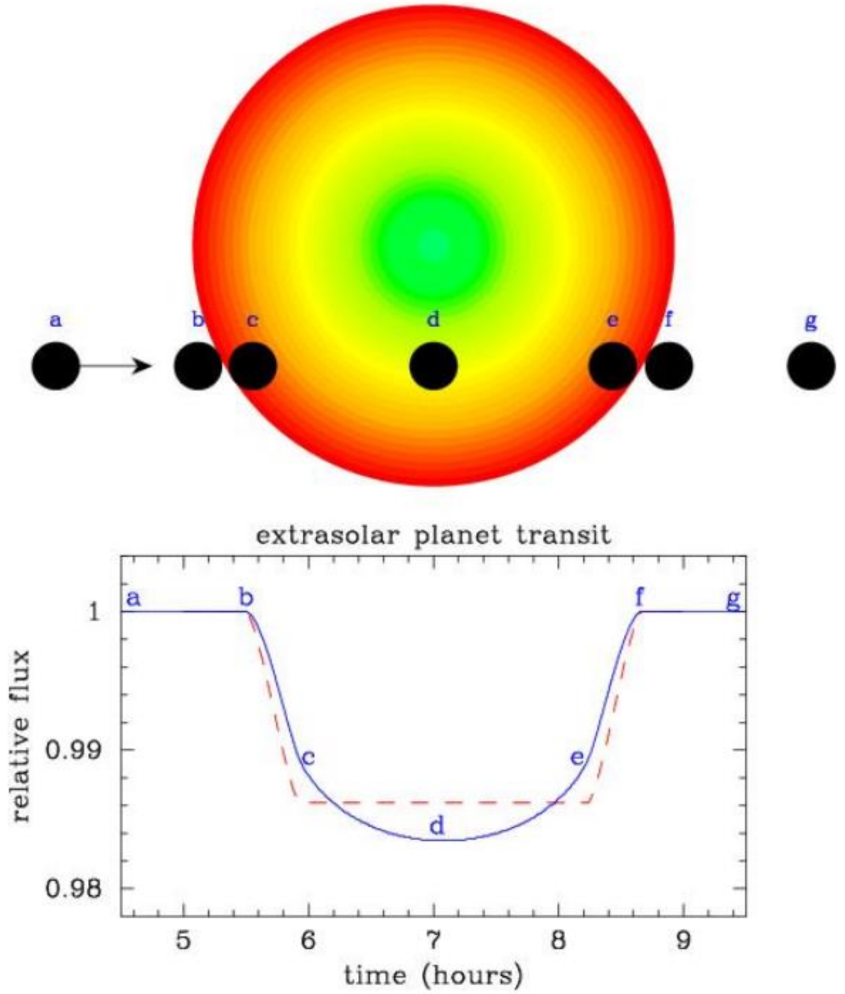
Examples

Limb darkening



Credit: PAR

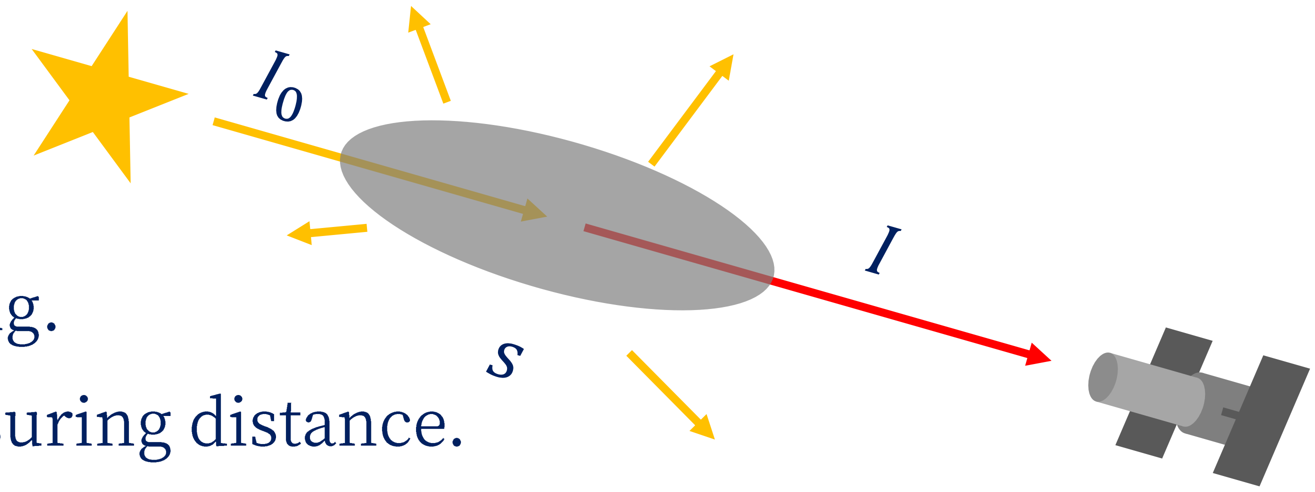

Brocken Inaglory



Coughlin 2012

Examples

Dust Extinction



Extinction = absorption + scattering.

This can be important in e.g. measuring distance.

The original distance modulus is:

$$m_\lambda - M_\lambda = 5 \log D - 5$$

But when there is dust, we should correct for its extinction

$$m_\lambda - M_\lambda = 5 \log D - 5 + A_\lambda$$

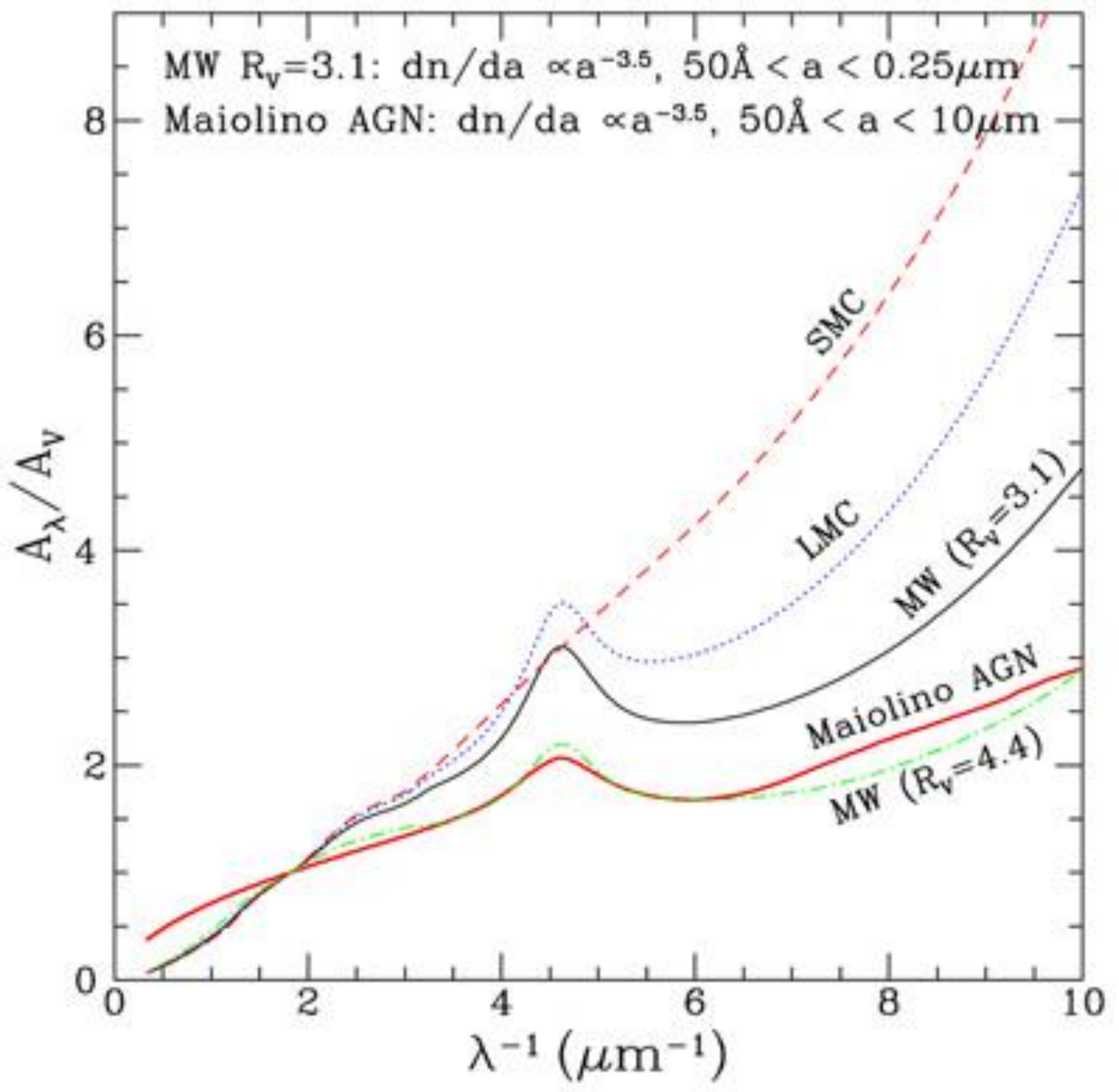
Examples

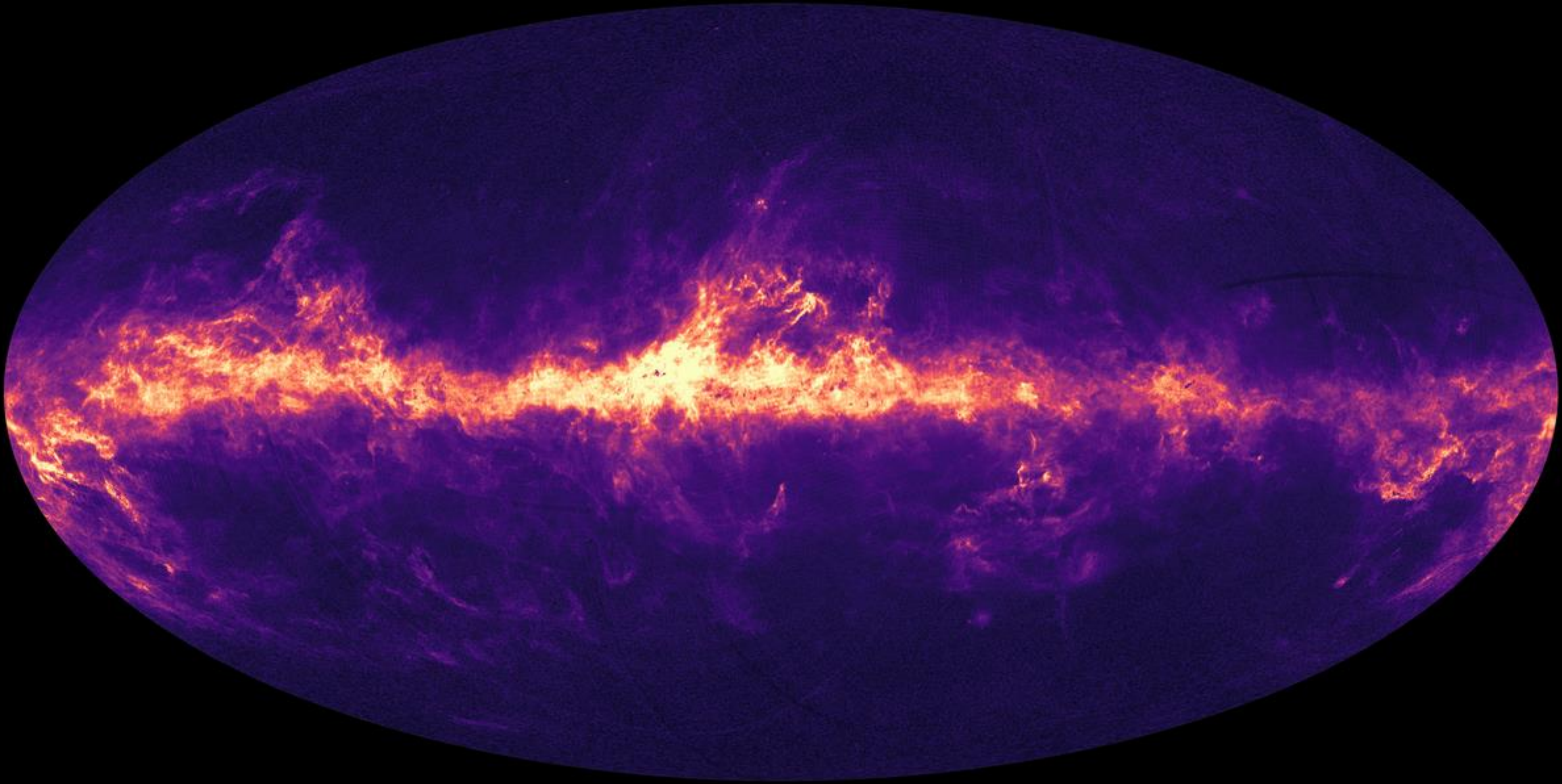
Dust Extinction

More importantly, extinction strength changes with wavelength.

In optical, the short wavelength light usually suffers stronger extinction, causing **reddening**.

The wavelength dependence of extinction is called **extinction curve**.





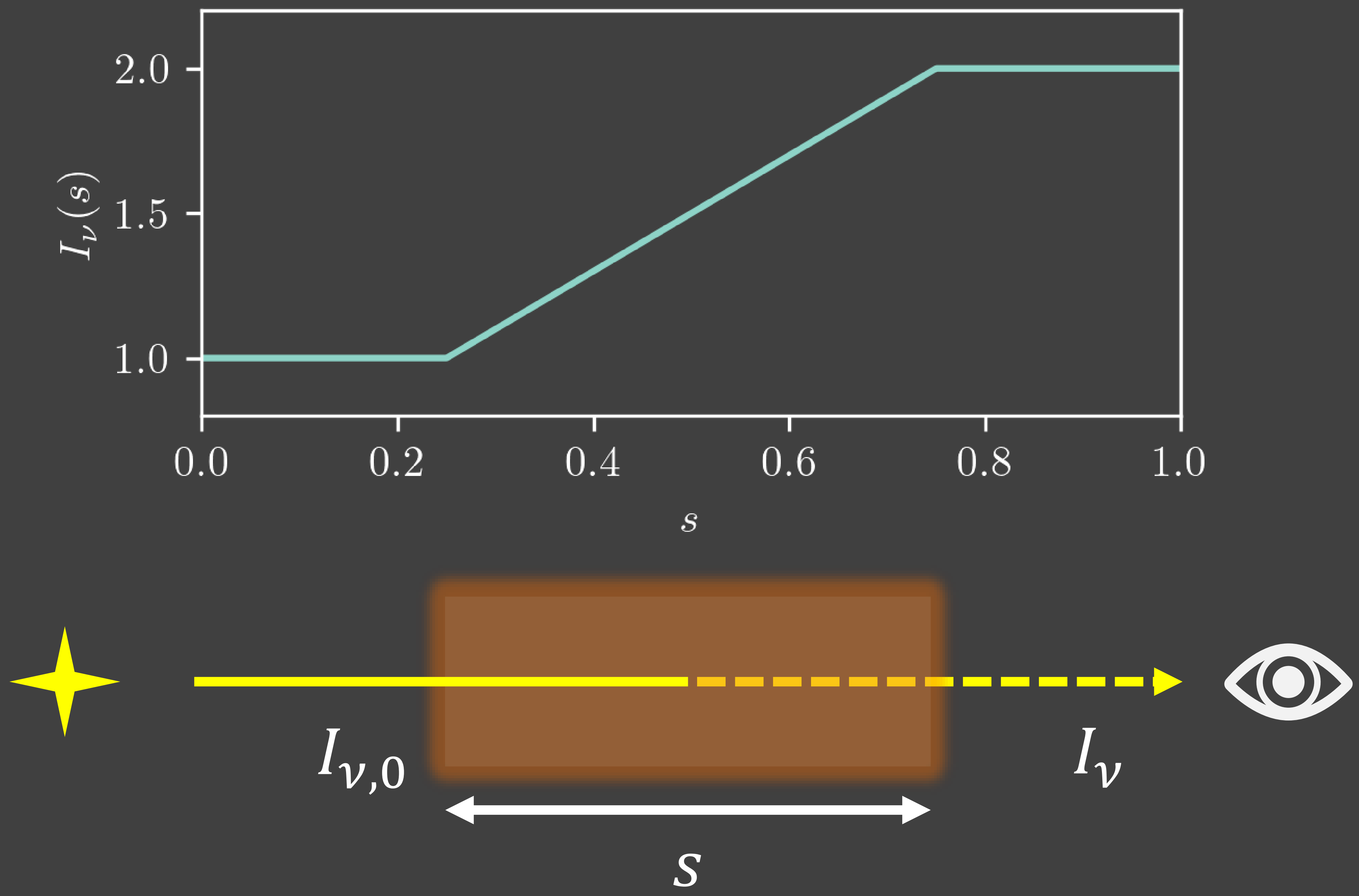
What about emission?

Since emission does not depends on the incident intensity*,

in pure emission case, we simply have

$$dI_\nu = j_\nu ds, \quad I_\nu = \int j_\nu ds$$

Where j_ν is called the **emission coefficient**.



2.0
1.5
1.0
$I_\nu(s)$
0.0
0.2
0.4
0.6
0.8
1.0
s
$I_{\nu,0}$
I_ν
s

The Radiative Transfer Equation (RTE)

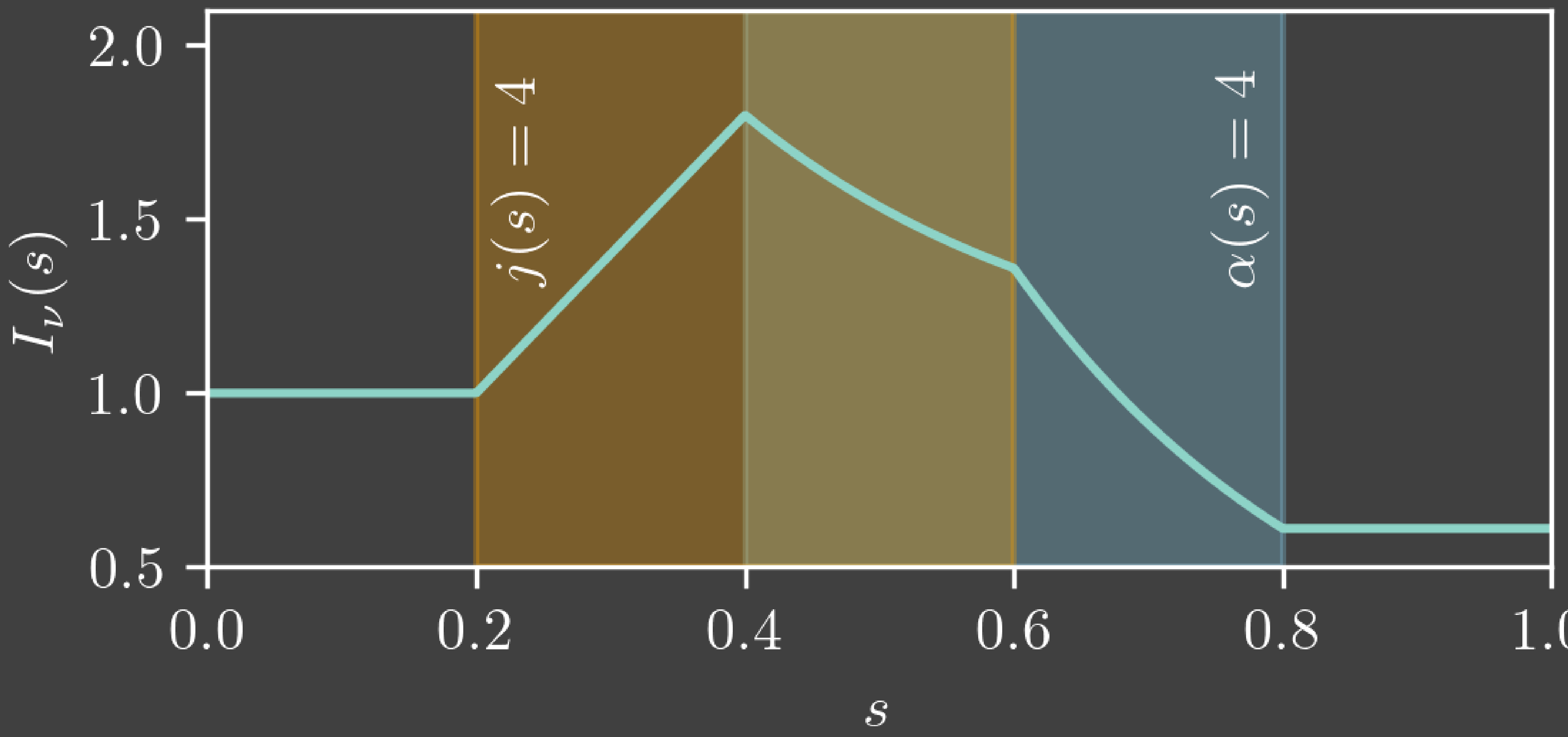
$$\frac{dI_\nu}{ds} = -\kappa_\nu \rho I_\nu + j_\nu$$

The change of specific intensity per unit length

The incident light that is absorbed

The newly emitted light

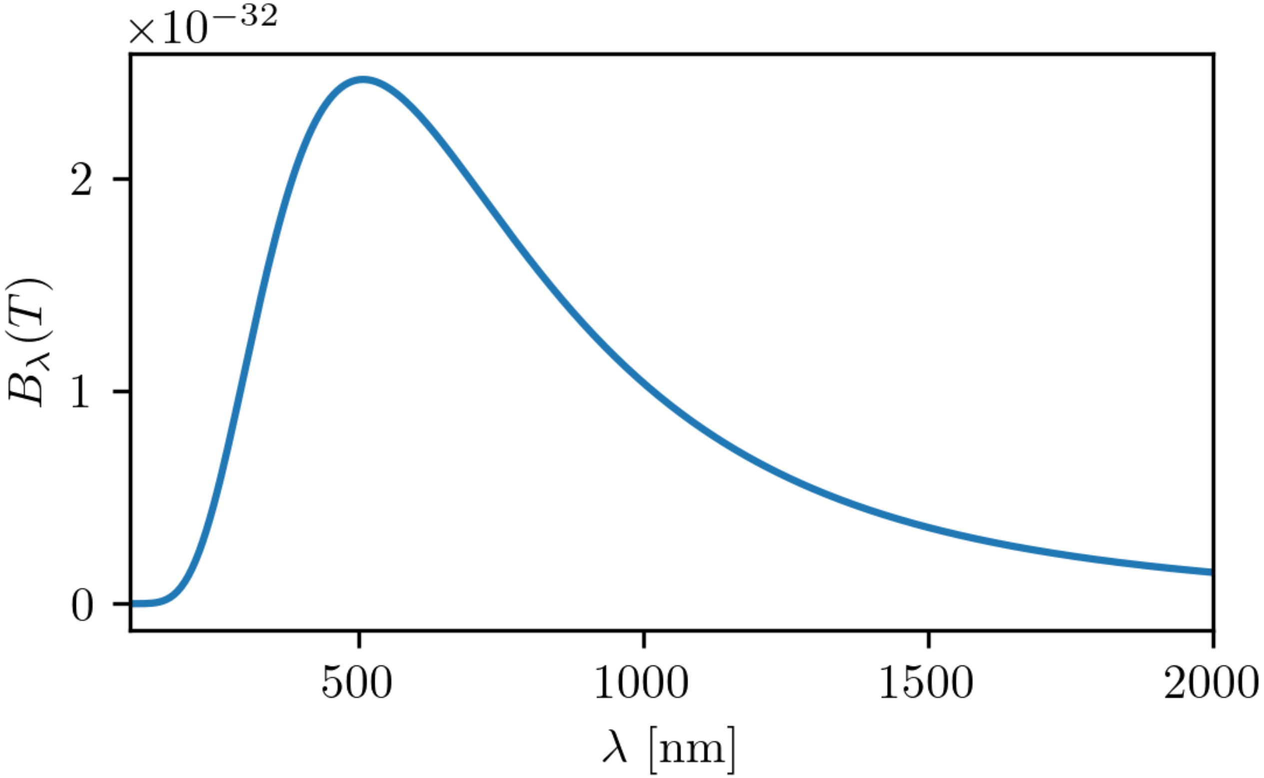
$$I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu(s_0,s)} + \int_{s_0}^{s} j_\nu(s')\, e^{-\tau_\nu(s',s)} ds'$$



Emission mechanisms

Black body radiation (BBR)

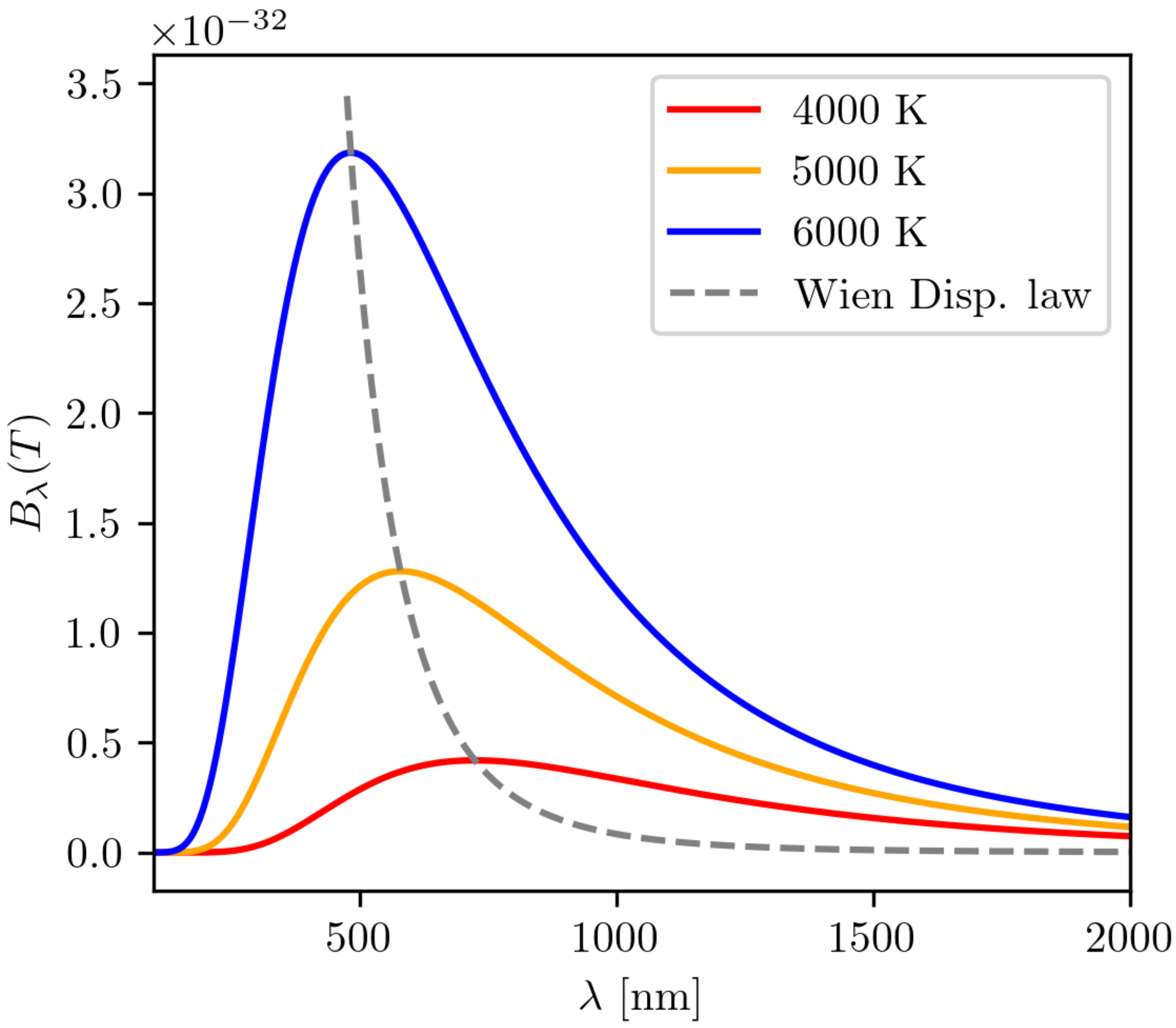
Emission coming from matter (and photons) in thermal equilibrium.



Planck function

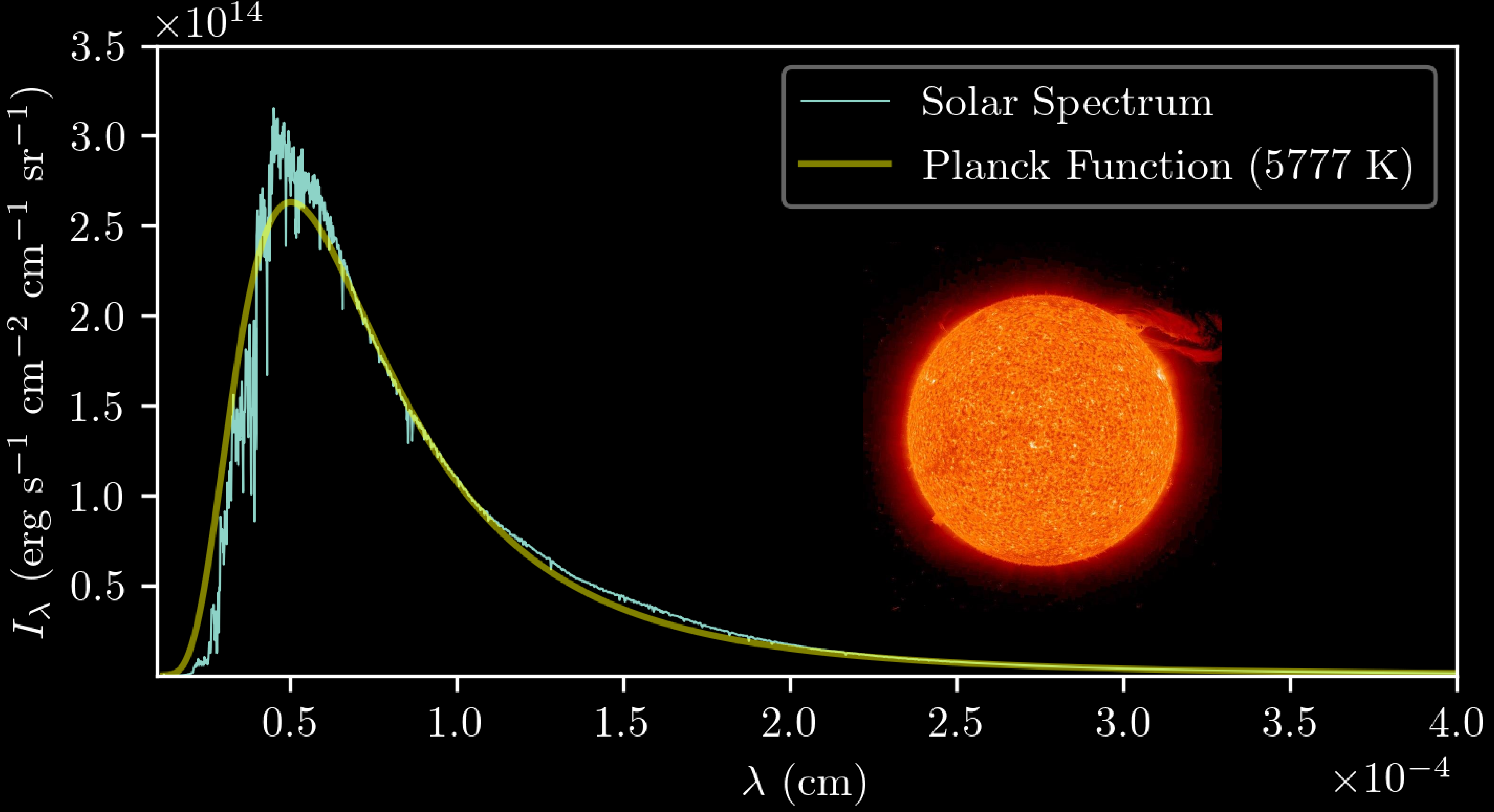
$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/(\lambda k_B T)} - 1}$$

$$B_\nu(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/(k_B T)} - 1}$$



Wien Displacement Law

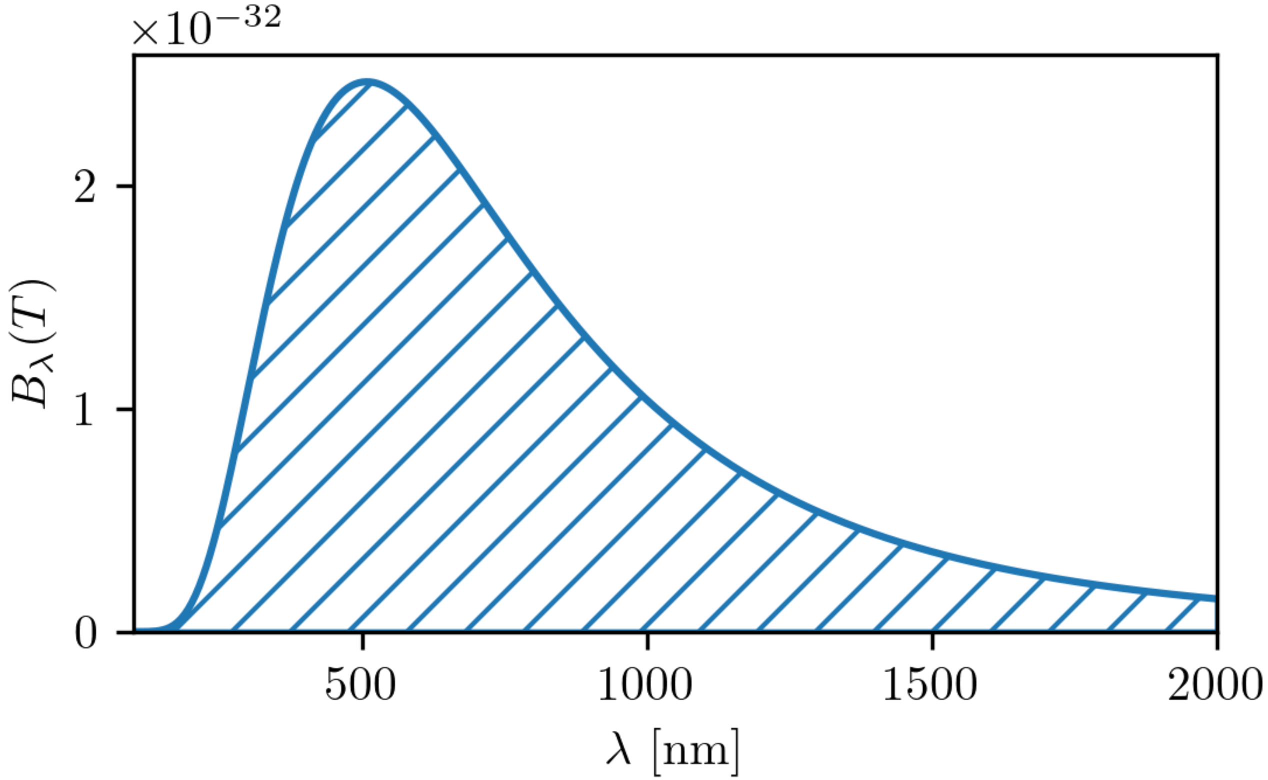
$$\lambda_{\max} = \frac{2.9 \times 10^{-3}}{T\ (K)}\ \mathrm{m}$$



×10^14
3.5
3.0
2.5
2.0
1.5
1.0
0.5
I_λ (erg s^−1 cm^−2 cm^−1 sr^−1)
Solar Spectrum
Planck Function (5777 K)
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
λ (cm)
×10^−4

Black body radiation (BBR)

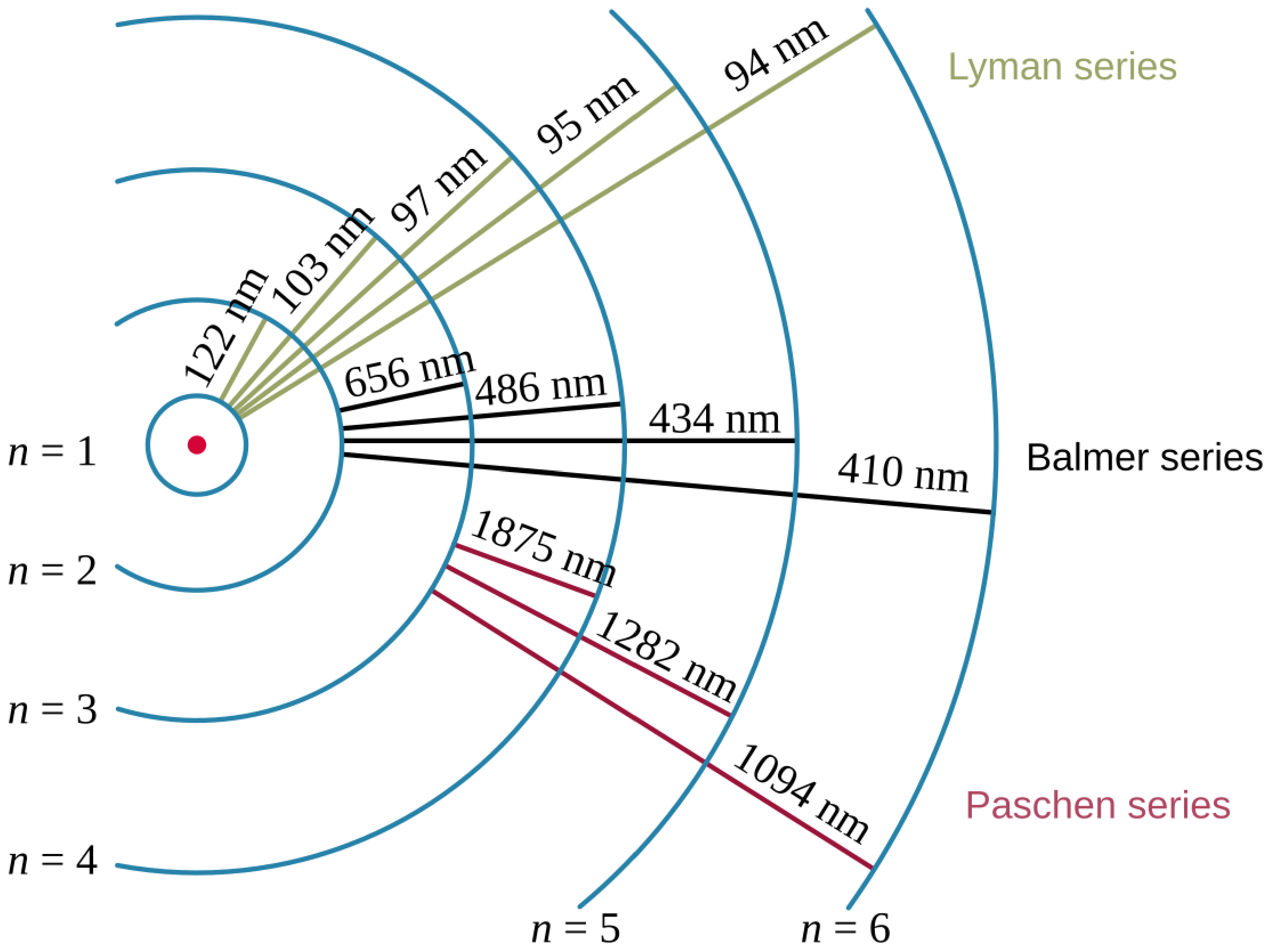
Total (all wavelength/frequency) flux coming from the black body.



Stefan-Boltzmann Law

$$F = \int B_\lambda d\lambda = \sigma_{\mathrm{SB}} T^4$$

$$L = 4\pi R^2 \sigma_{\mathrm{SB}} T^4$$



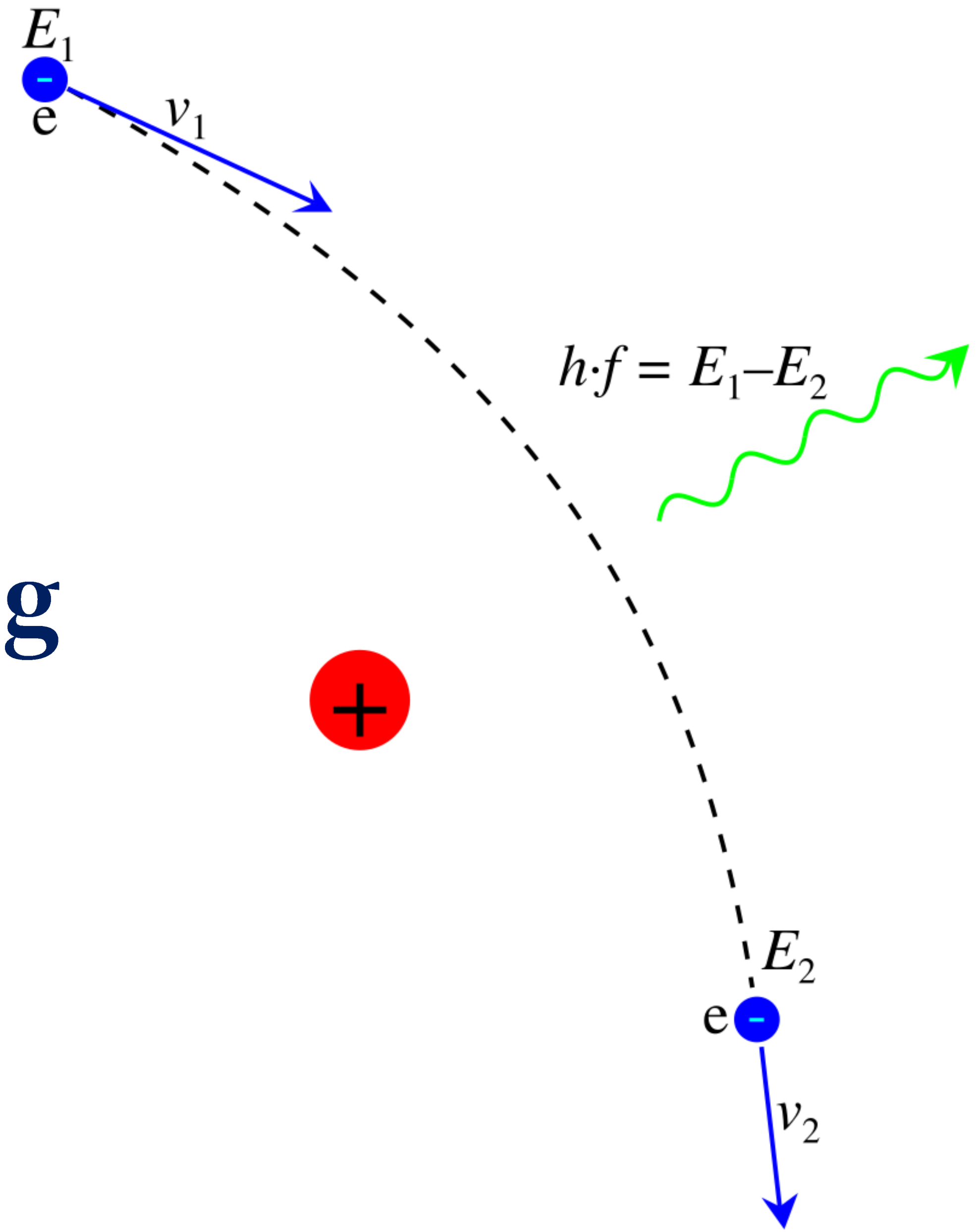
Emission mechanisms

Electron transitions

Szdori, OrangeDog

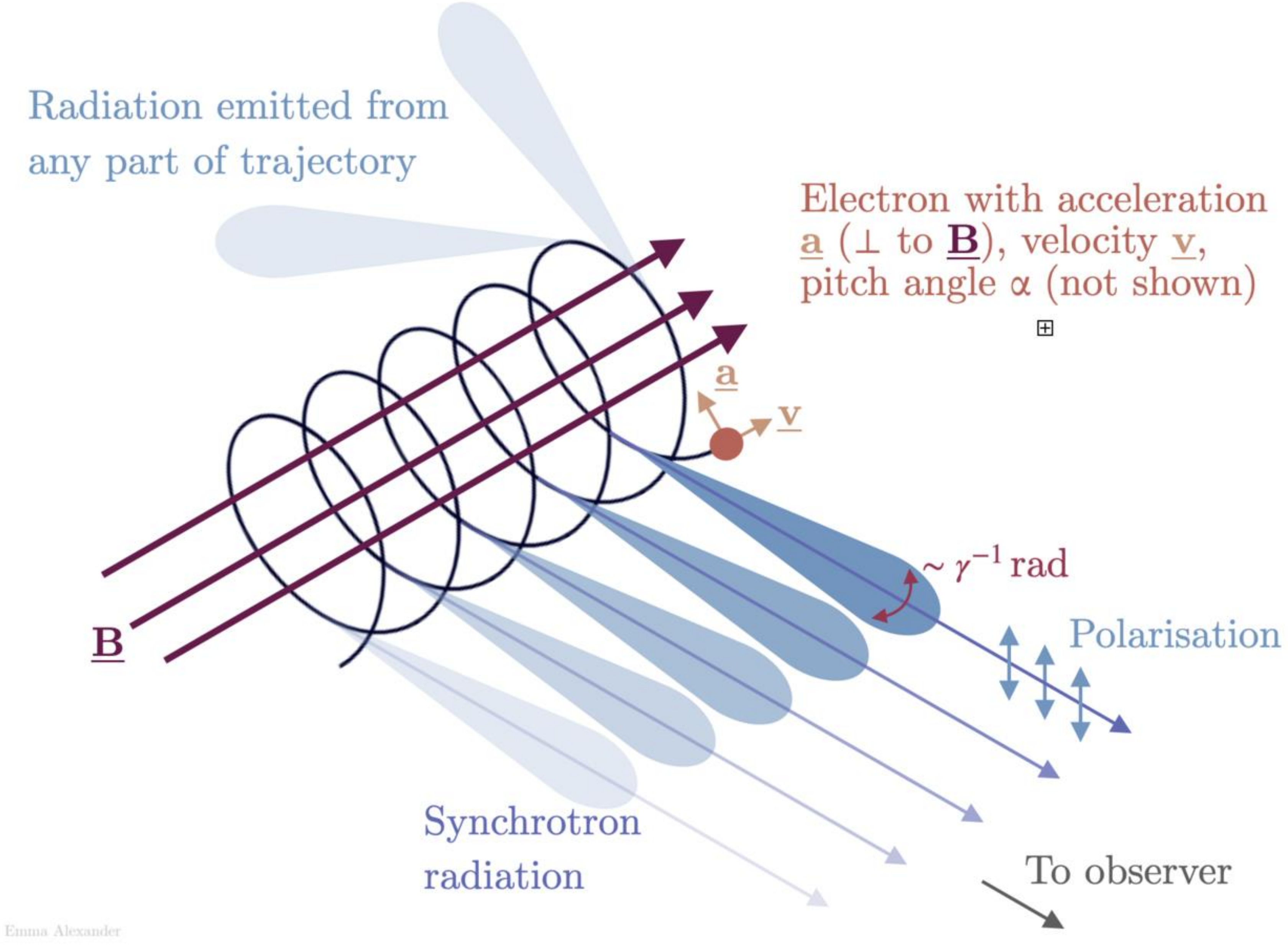
Emission mechanisms

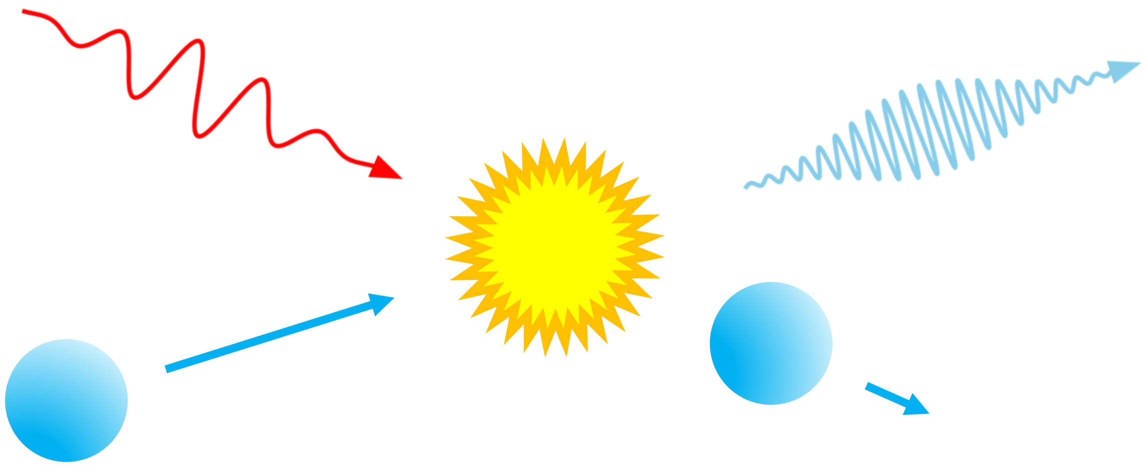
Free-free / Bremsstrahlung radiation



Emission mechanisms

Synchrotron radiation





Emission mechanisms

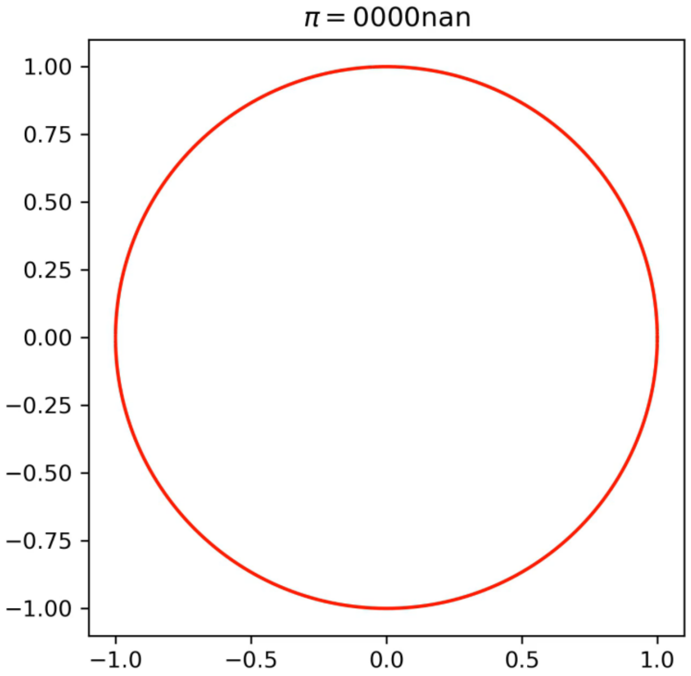
Inverse Compton scattering

Complicated situations

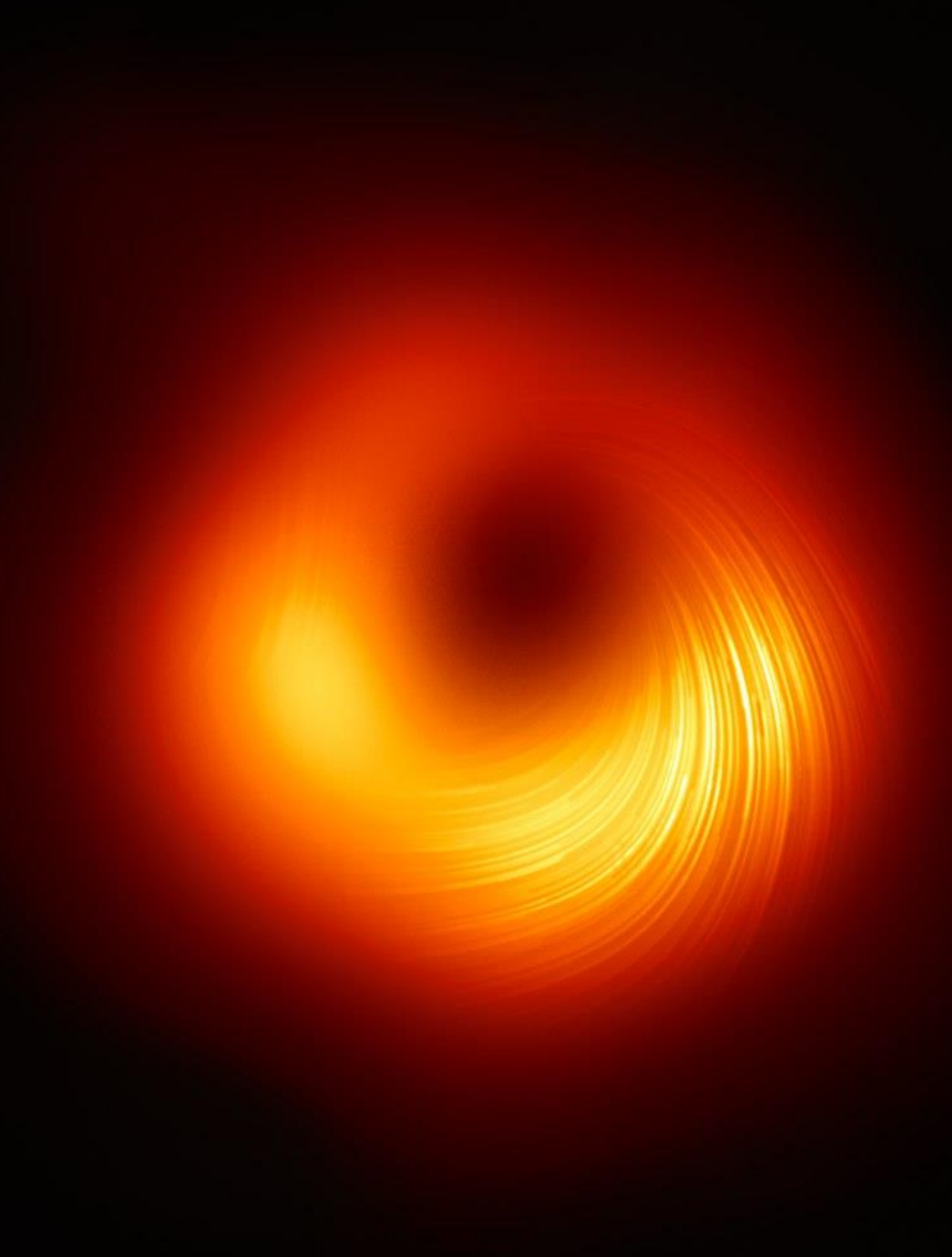
Computer go brrrrrrrrrr

Numerical Radiative Transfer

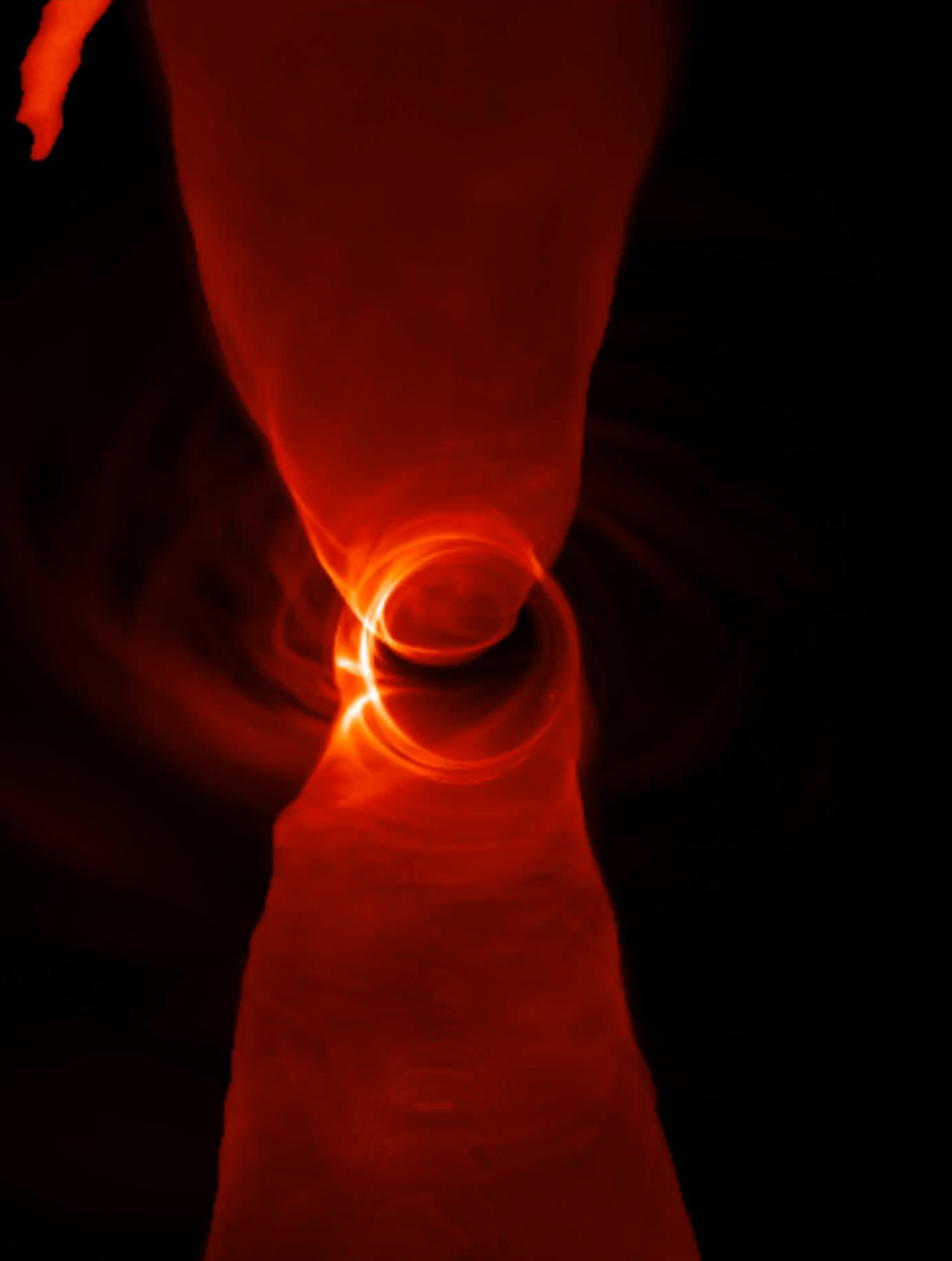
Utilizing **Monte-Carlo method** and **Ray Tracing** to solve RTE.



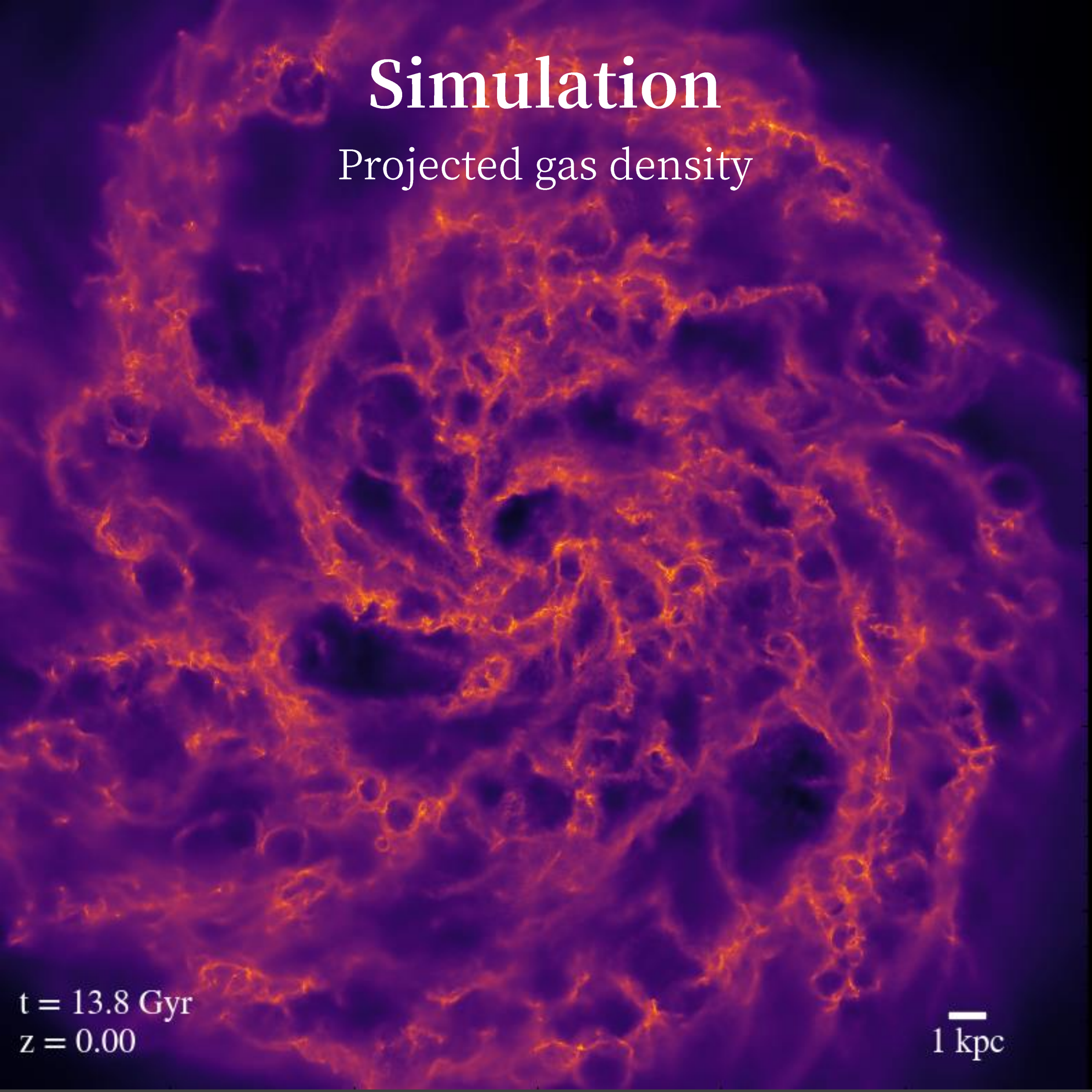
GR ray tracing

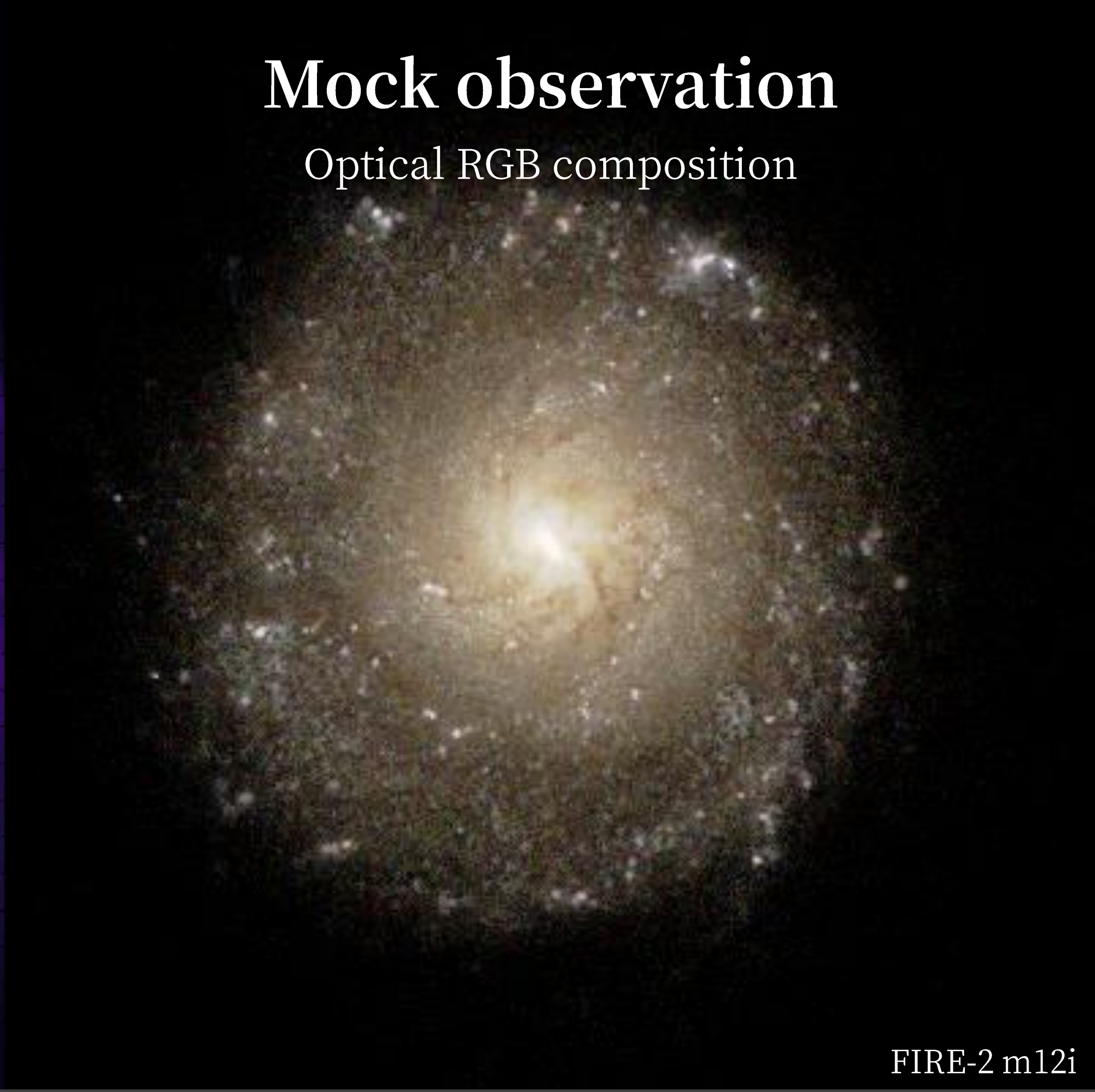


EHT Collaboration



Lia Medeiros, Chi-Kwan Chan, Feryal Özel, Dimitrios Psaltis





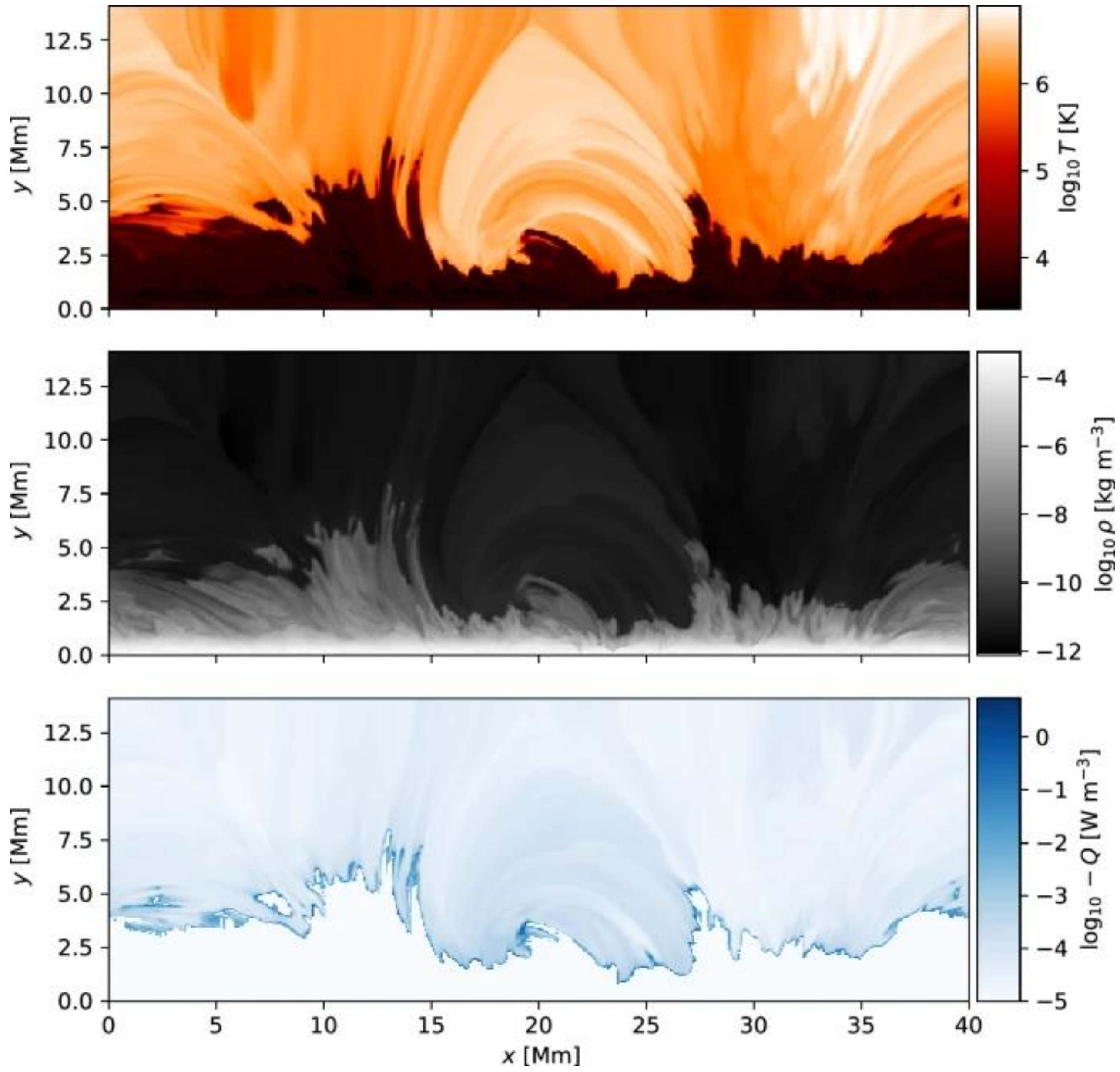
Better/direct comparison with observations

Computer go brrrrrrrrrr

Radiative Hydrodynamics

Important in e.g. stellar atmosphere, super-Eddington accretion disk.

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla} \cdot (\rho \mathbf{v})$$
$$\frac{\partial \mathbf{p}}{\partial t} = -\boldsymbol{\nabla} \cdot (\mathbf{v} \otimes \mathbf{p} - \boldsymbol{\tau}) - \boldsymbol{\nabla} P + \mathbf{J} \times \mathbf{B} + \rho \mathbf{g} - \boldsymbol{\nabla} \mathbf{P}_{\mathrm{rad}}$$
$$\frac{\partial e}{\partial t} = -\boldsymbol{\nabla} \cdot (e \mathbf{v}) - P \boldsymbol{\nabla} \cdot \mathbf{v} + Q + Q_{\mathrm{rad}}$$



Jorrit Leenaarts (2021)

Summary

- In astrophysics, we often use **specific intensity** [J m^{-2} sr^{-1} s^{-1} Hz^{-1}] to describe the strength of light, which does not decay with distance.
- Specific intensity is changed by **absorption**, **scattering** and **emission**, described by **radiative transfer equation**.
- Radiative transfer effects are often discussed using **optical depth**.
- There are many mechanisms that generates/absorb radiation.
- Complicated radiative transfer problems are solved numerically.