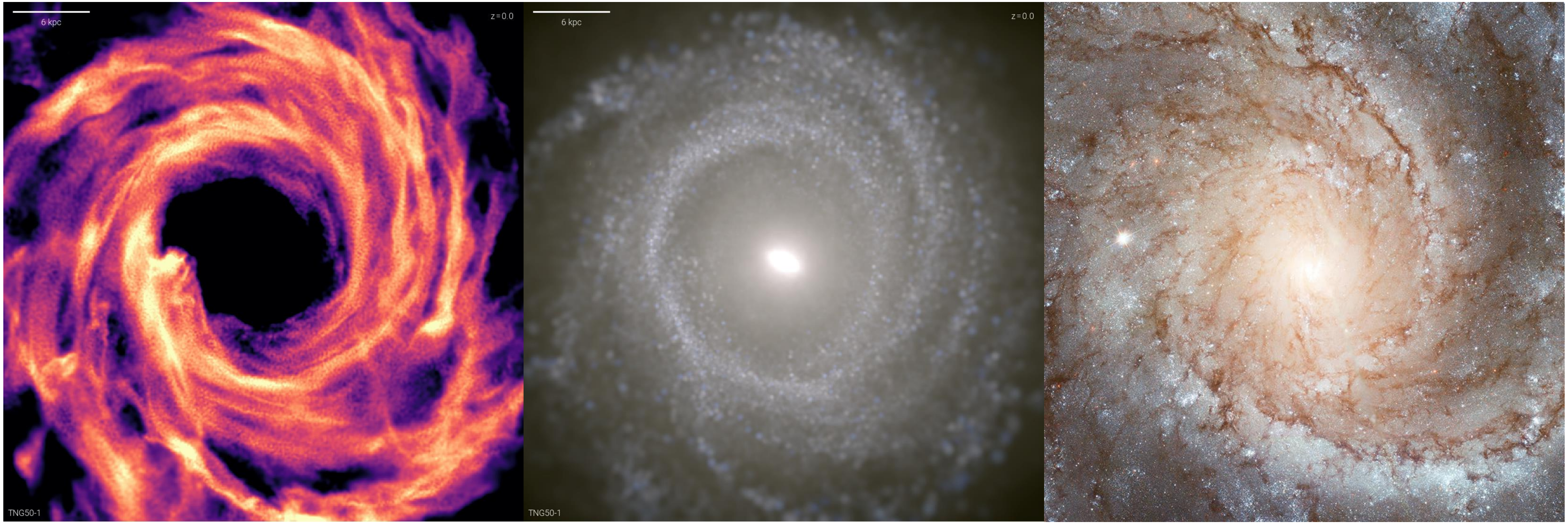
Radiative Transfer

2022.02.22 AstroRead @ NTHU Astronomy Club

Why do we care about radiative processes?

為什麼我們要學輻射過程？



illustrisTNG | NASA, ESA, K. Kuntz, F. Bresolin, J. Trauger, J. Mould, Y.-H. Chu, Canada-France-Hawaii Telescope/J.-C. Cuillandre/Coelum, and G. Jacoby, B. Bohannan, and M. Hanna/NOAO/AURA/NSF

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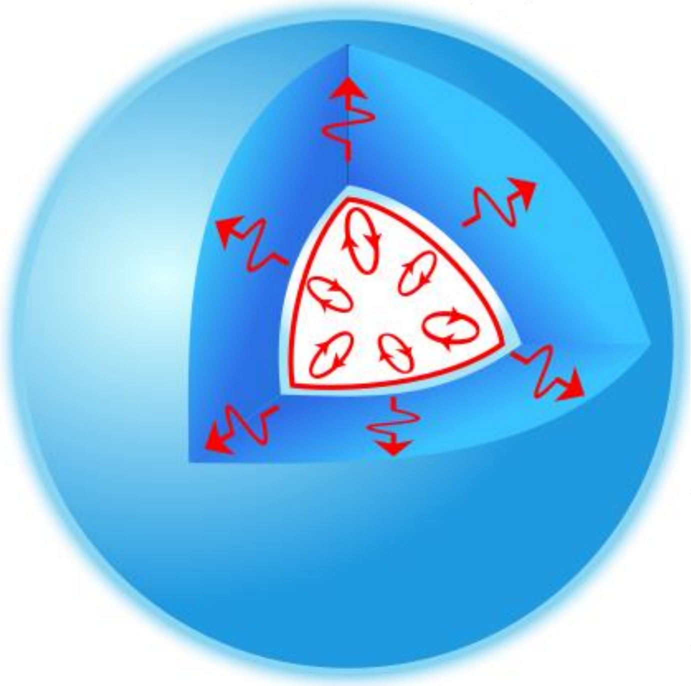

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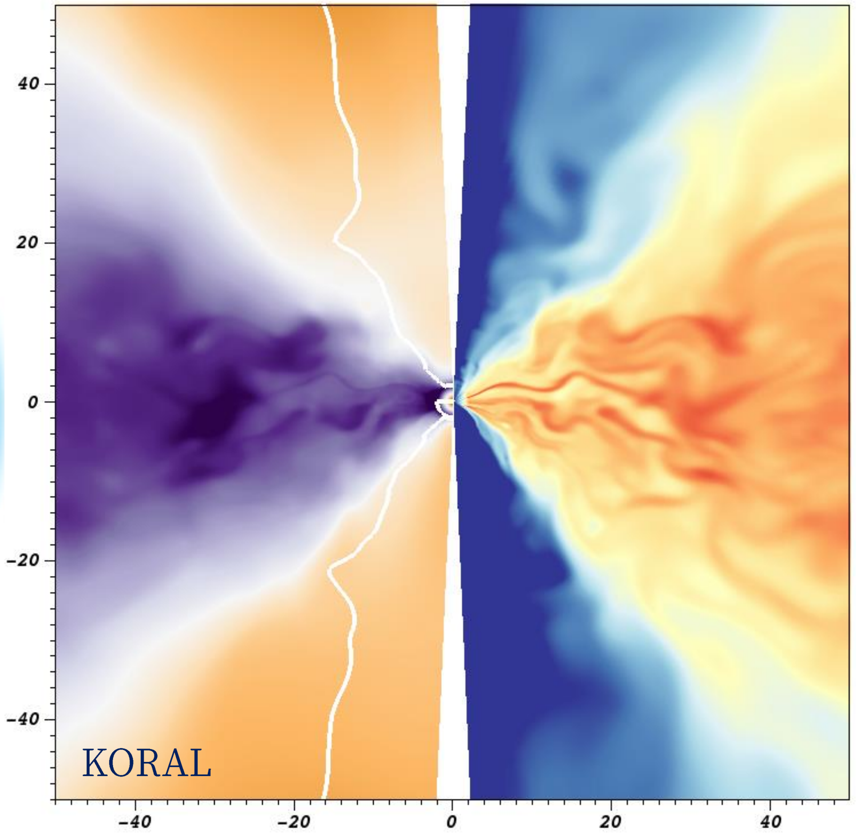
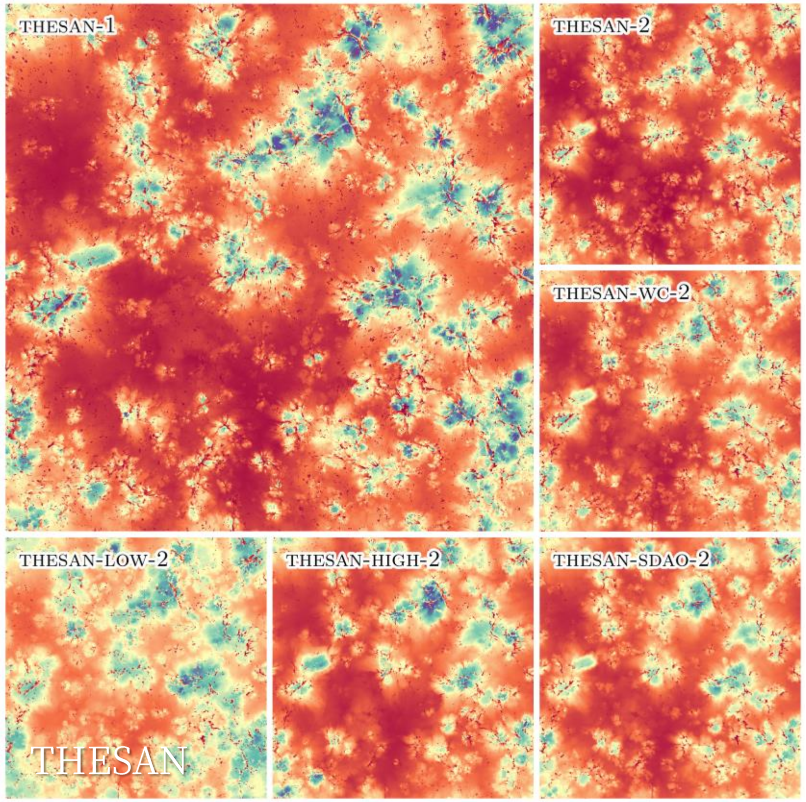




How to define the **Intensity** of light?

描述光的強度

「亮」的天體到底是什麼意思？

In physical sciences [edit]

Physics [edit]

- Intensity (physics), power per unit area (W/m^2)
- Field strength of electric, magnetic, or electromagnetic fields (V/m, T, etc.)
- Intensity (heat transfer), radiant heat flux per unit area per unit solid angle (W·m^{-2}·sr^{-1})
- Electric current, whose value is sometimes called *current intensity* in older books

Optics [edit]

- Radiant intensity, power per unit solid angle (W/sr)
- Luminous intensity, luminous flux per unit solid angle (lm/sr or cd)
- Irradiance, power per unit area (W/m^2)

Astronomy [edit]

- Radiance, power per unit solid angle per unit projected source area (W·sr^{-1}·m^{-2})

Seismology [edit]

- Mercalli intensity scale, a measure of earthquake impact
- Japan Meteorological Agency seismic intensity scale, a measure of earthquake impact
- Peak ground acceleration, a measure of earthquake acceleration (g or m/s^2)

Acoustics [edit]

- Sound intensity, sound power per unit area

國際單位制的輻射量單位 閱·論·編

物理量	符號	國際單位制	單位符號	注釋
輻射出射度（Radiant exitance）	M_e	瓦特每平方公尺	W·m^{-2}	表面出射的輻射通量
輻射度（Radiosity）	J_e or $J_{e\lambda}$	瓦特每平方公尺	W·m^{-2}	表面出射及反射的輻射通量總和
輻射率（Radiance）	L_e	瓦特每立弳每平方公尺	W·sr^{-1}·m^{-2}	每單位立體角每單位投射表面的輻射通量。
輻射能（Radiant energy）	Q_e	焦耳	J	能量。
輻射能量密度（Radiant energy density）	ω_e	焦耳每立方公尺	J·m^{-3}	
輻射強度（Radiant intensity）	I_e	瓦特每立弳	W·sr^{-1}	每單位立體角的輻射通量。
輻射曝光量（Radiant exposure）	H_e	焦耳每平方公尺	J·m^{-2}	
輻射通量（Radiant flux）	Φ_e	瓦特	W	每單位時間的輻射能量，亦作「輻射功率」。
輻照度（Irradiance）	E_e	瓦特每平方公尺	W·m^{-2}	入射表面的輻射通量。
光譜輻射出射度（Spectral radiant emittance）	$M_{e\lambda}$ 或 $M_{e\nu}$	瓦特每立方公尺 *或* 瓦特每平方公尺每赫茲	W·m^{-3} *or* W·m^{-2}·Hz^{-1}	表面出射的輻射通量的波長或頻率的分布
光譜輻射率（Spectral radiance）	$L_{e\lambda}$ 或 $L_{e\nu}$	瓦特每立弳每立方公尺 或 瓦特每立弳每平方公尺每赫茲	W·sr^{-1}·m^{-3} 或 W·sr^{-1}·m^{-2}·Hz^{-1}	常用W·sr^{-1}·m^{-2}·nm^{-1}
光譜輻照度（Spectral irradiance）	E_λ 或 E_ν	瓦特每立方公尺 *或* 瓦特每平方公尺每赫茲	W·m^{-3} *或* W·m^{-2}·Hz^{-1}	通常測量單位為 W·m^{-2}·nm^{-1}
光譜功率（Spectral power）	$\Phi_{e\lambda}$	瓦特每米	W·m^{-1}	輻射通量的波長分布
光譜強度（Spectral intensity）	$I_{e\lambda}$	瓦特每立弳每米	W·sr^{-1}·m^{-1}	輻射強度的波長分布

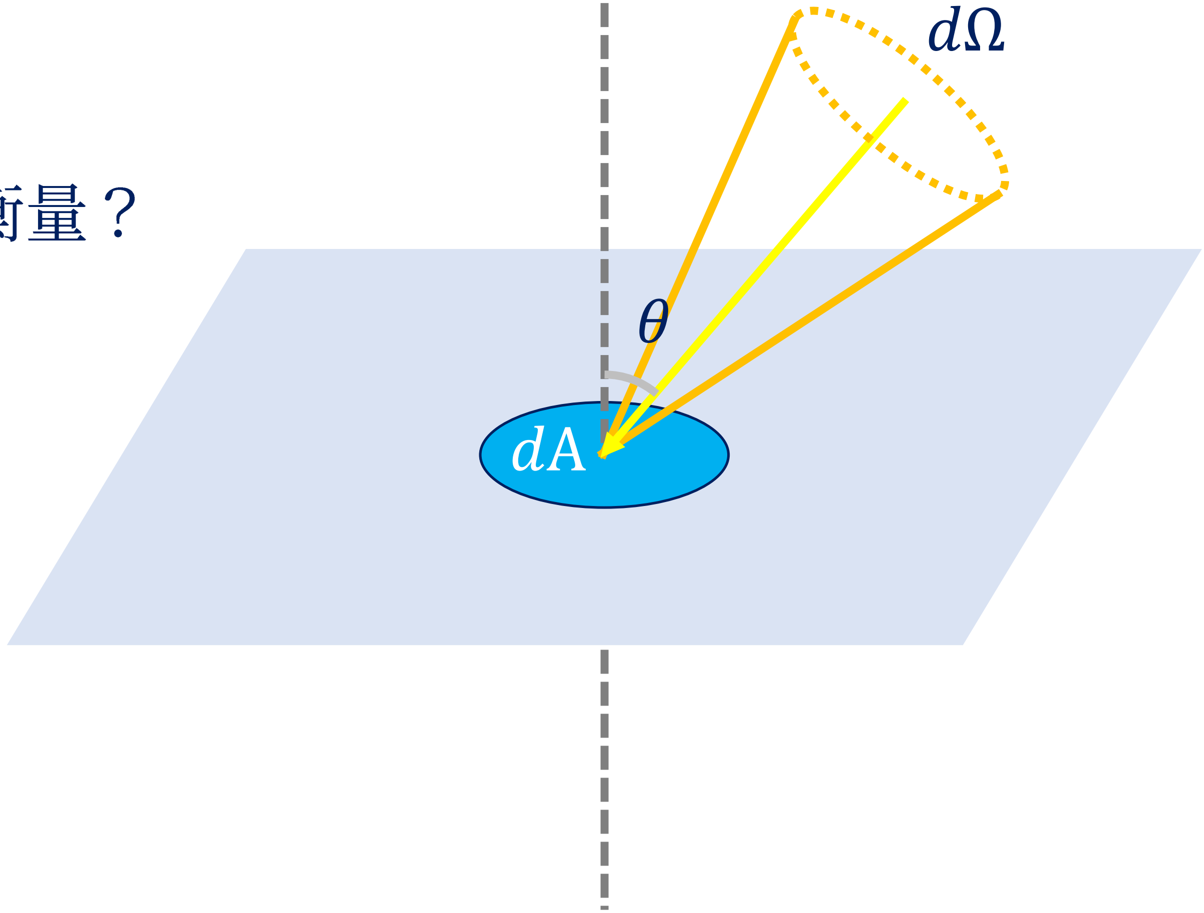
How to define the **Intensity** of light?

描述光的強度・續

回歸本質，天體的亮應當用哪個物理量衡量？

又有什麼因素會影響天體的「亮度」？

$$\frac{dE}{\cos\theta\, dt dA d\Omega d\nu} = I_\nu$$



Specific Intensity

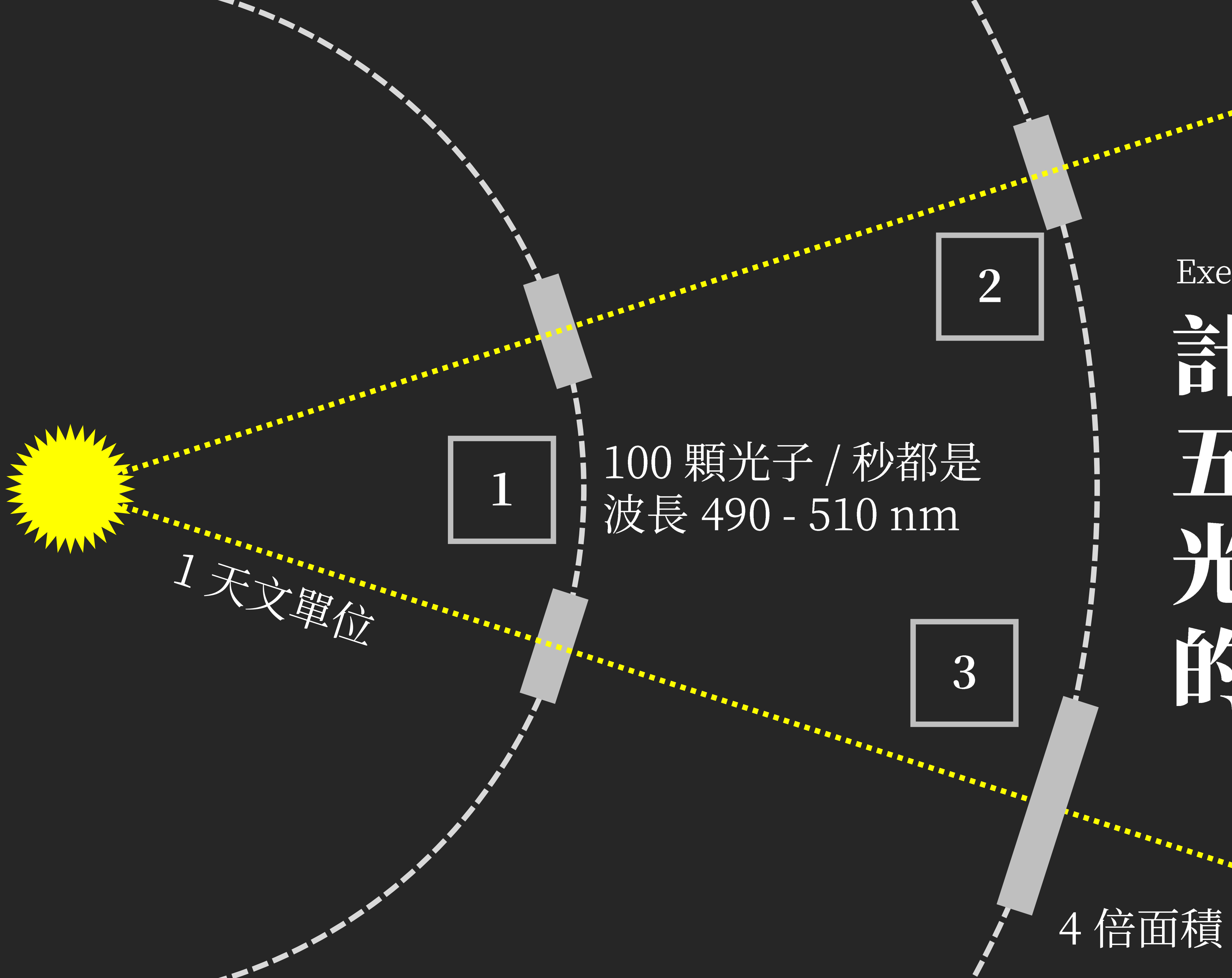
描述光的強度・再

$$\frac{dE}{\cos\theta \, dt dA d\Omega d\nu} = I_\nu$$

1. Energy Received (E)，單位 [erg]
2. Total Flux (F)，單位 [erg s^{-1}]
3. Flux (f)，單位 [erg cm^{-2} s^{-1}]
4. Total Intensity (I)，單位 [erg cm^{-2} sr^{-1} s^{-1}]，又叫 Surface Brightness
5. Specific Intensity (I_ν)，單位 [erg cm^{-2} sr^{-1} s^{-1} Hz^{-1}]

The **names** are not important. What you should care about is the **units.**

I_ν is convenient because it is an **intrinsic** property of the source.



Exercise

計算上述五個描述光強度的物理量

直覺法：

[1]

- $E = 100h\nu$
- $F = 100h\nu/t$
- $f = 100h\nu/At$
- $I = 100h\nu/At\Omega$
- $I_\nu = 100h\nu/At\Omega\Delta\nu$

[2]

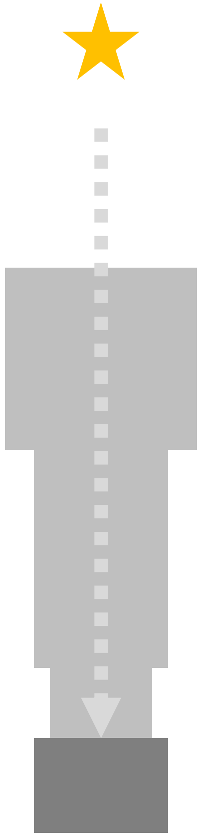
- $E = 25h\nu$
- $F = 25h\nu/t$
- $f = 25h\nu/At$
- $I = 25h\nu/At\left(\frac{\Omega}{4}\right)$
- $I_\nu = \frac{25h\nu}{At\left(\frac{\Omega}{4}\right)\Delta\nu}$

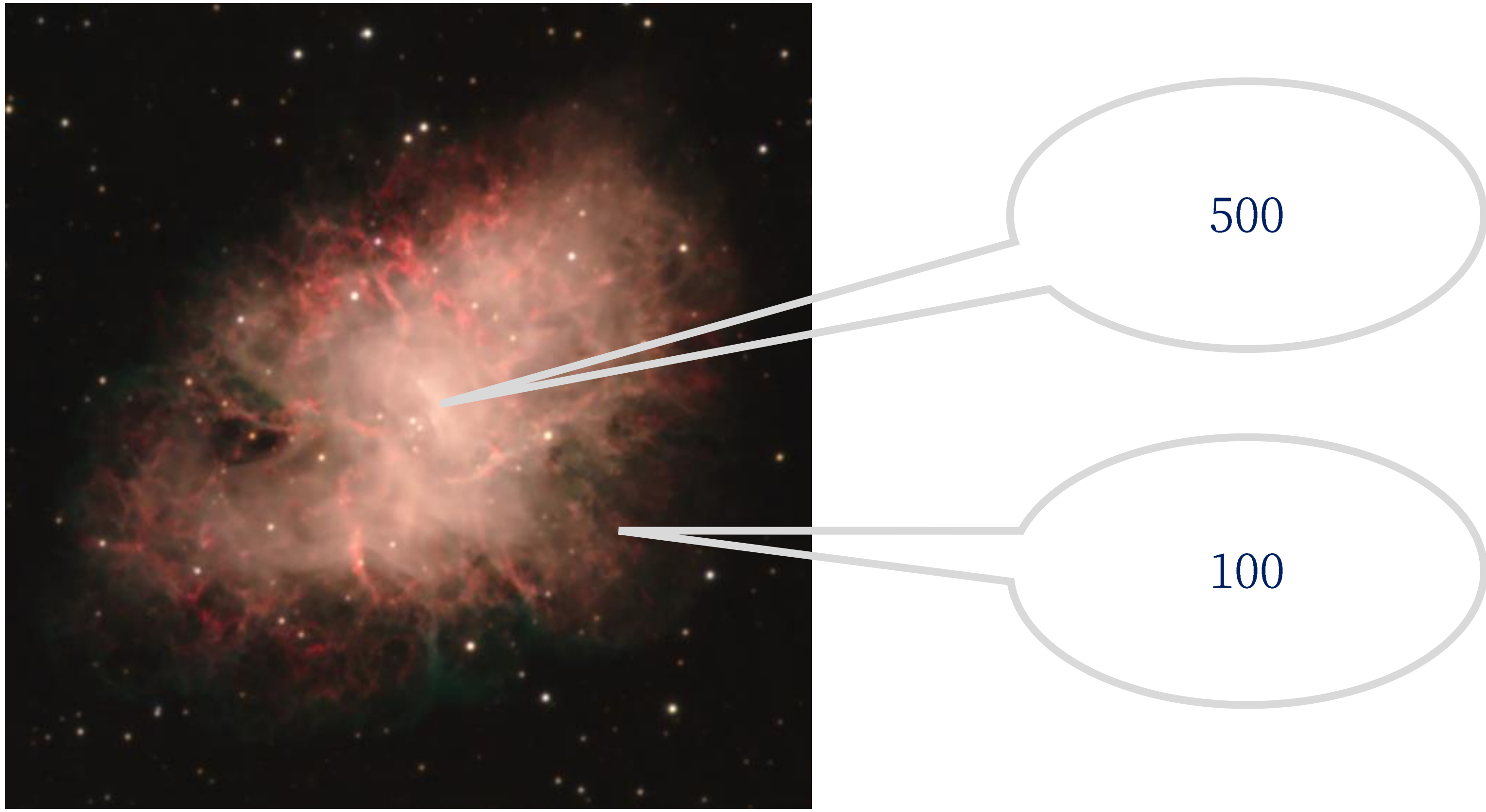
[3]

- $E = 100h\nu$
- $F = 100h\nu/t$
- $f = 100h\nu/(4A)t$
- $I = \frac{100h\nu}{(4A)t\left(\frac{\Omega}{4}\right)}$
- $I_\nu = \frac{100h\nu}{4At\left(\frac{\Omega}{4}\right)\Delta\nu}$

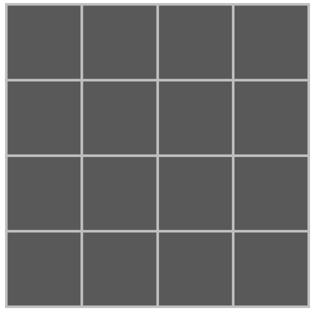
Exercise

感光元件的 Count 對應的物理量是？





Credit: 許淵明



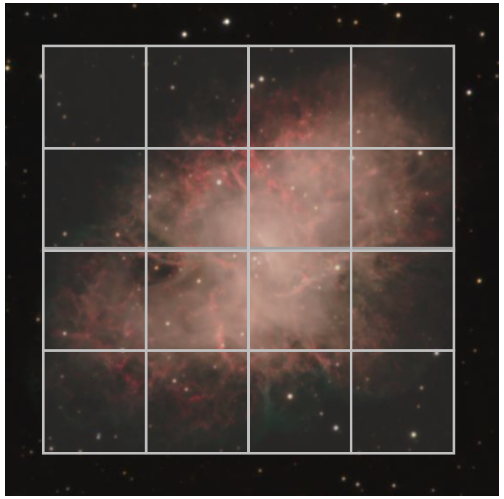
導出參數

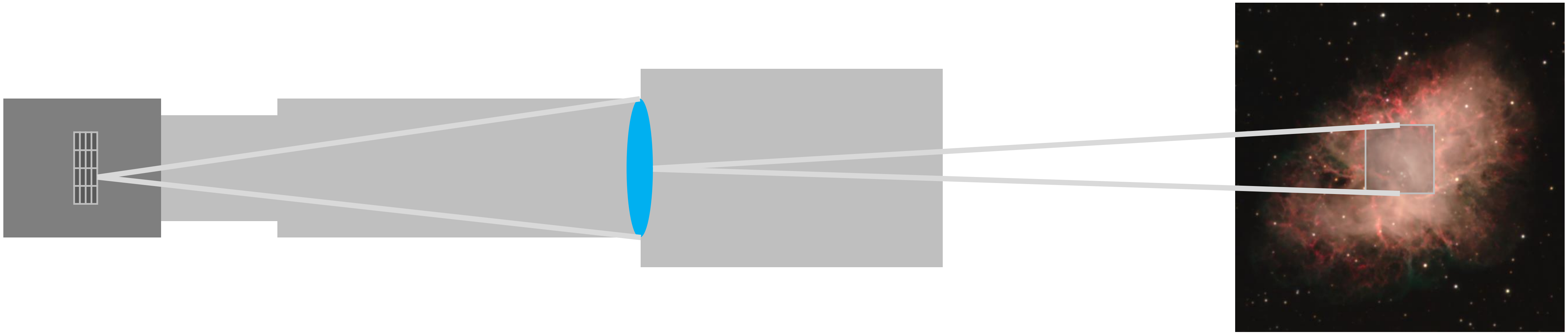
像素數量：$n = (L/\ell)^2$

焦比：$f = F/D$

視野：$\mathrm{FOV} = (L/F)^2$

單像素視野（空間解析度）：$\Omega_p = (\ell/F)^2 = \mathrm{FOV}/n$





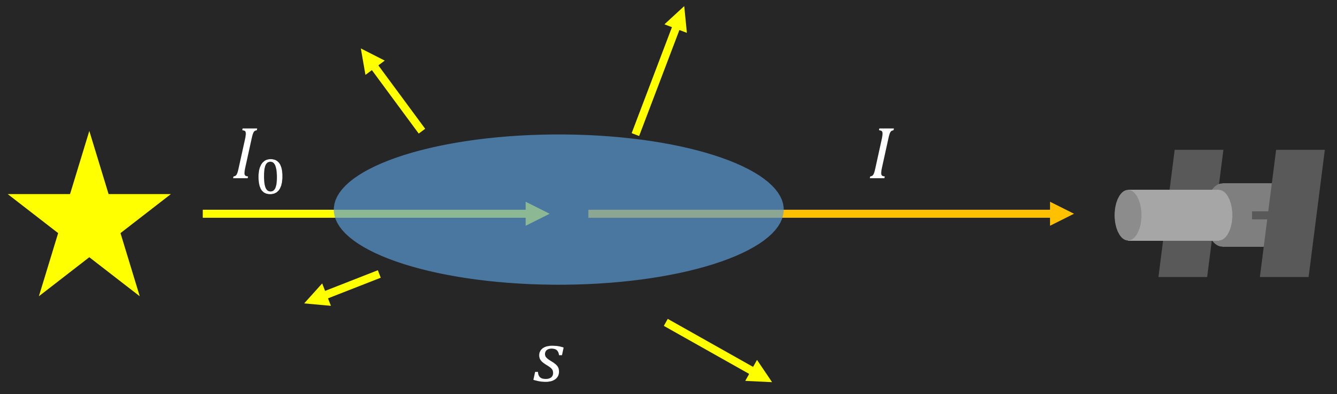
$$N = \int_T \int_A \int_\Omega I\,d\Omega\,dA\,dt = I\Omega AT = I\left(\frac{\ell}{F}\right)^2 D^2 T = I\left(\frac{D}{F}\right)^2 \ell^2 T$$

Radiative Transfer

光的強度變化：輻射轉移

What will change the intensity of light?

Emission, **Absorption** and **Scattering**.



$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu$$

輻射轉移・續

More on absorption coefficient:

$$\alpha_\nu = n\sigma_\nu = \rho\kappa_\nu$$

With **no emission**, the radiation transfer equation reduce to:

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$

Solving this ODE we see

$$I_\nu(s) = I_\nu(0)\exp\left(-\int_0^s \alpha_\nu(s)ds\right) \equiv I_\nu(0)e^{-\tau_\nu}, \qquad d\tau_\nu = \alpha_\nu ds$$

We define **optical depth** accordingly.

Radiative Transfer

輻射轉移・再

On the other hand, if there is no absorption, we have

$$\frac{dI_\nu}{ds} = j_\nu \Rightarrow I_\nu(s) = I_\nu(0) + \int_0^s j_\nu ds$$

Which is kind of trivial. :p

Radiative Transfer

輻射轉移・改

For some reason, people often use another form of radiative transfer equ.

We define the **Source Function**:

$$S_\nu = \frac{j_\nu}{\alpha_\nu}$$

Plug it back into the R.T.E we get

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

Radiative Transfer

輻射轉移・改二

For some reason, people often use another form of radiative transfer equ.

We define the **Source Function**:

$$S_\nu = \frac{j_\nu}{\alpha_\nu}$$

Plug it back into the R.T.E, and use tau as spatial coordinate, we get

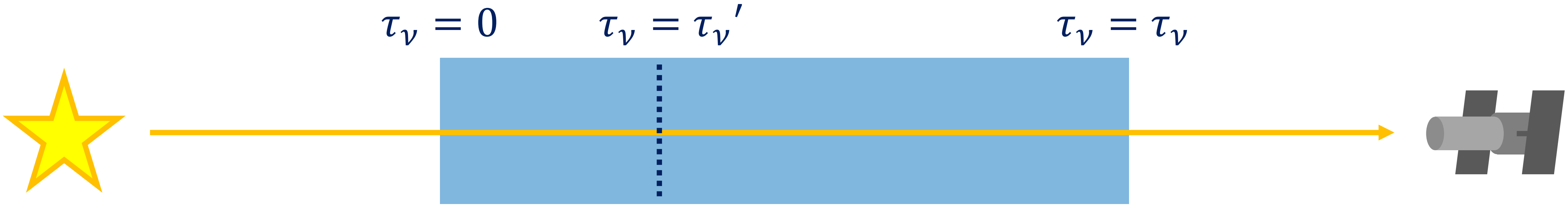
$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

輻射轉移・改三

In this form, there is an exact solution

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu$$

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(\tau_\nu')e^{-(\tau_\nu - \tau_\nu')}\mathrm{d}\tau_\nu'$$



Radiative Transfer

輻射轉移・改四

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(\tau'_\nu)e^{-(\tau_\nu-\tau'_\nu)}\mathrm{d}\tau'_\nu$$

When the source function and absorption is constant, the solution is

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

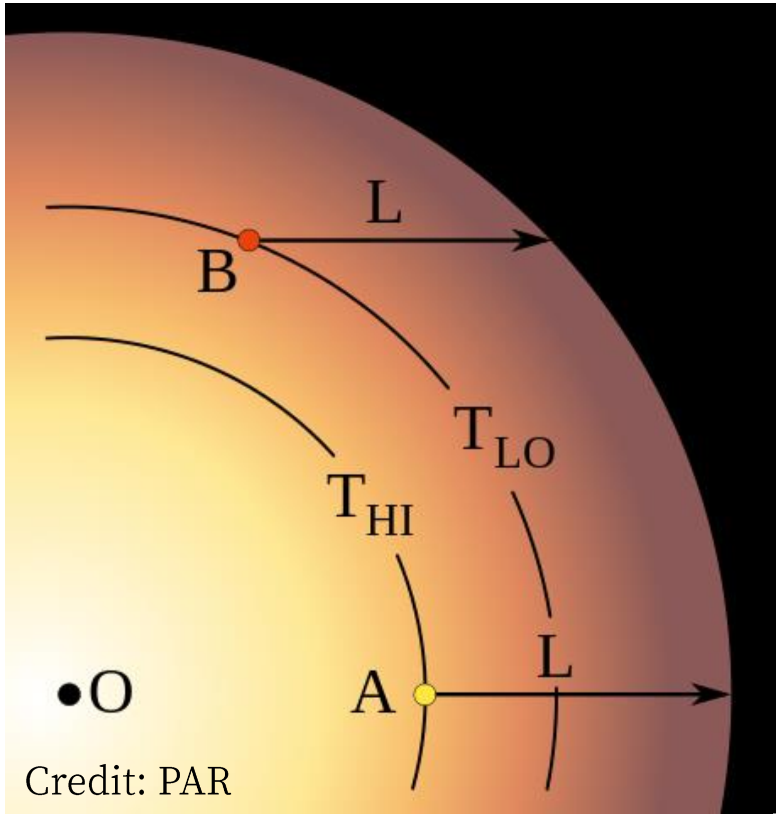
In some extreme cases:

- $\tau_\nu \to \infty \Rightarrow I_\nu \to S_\nu$
- $\tau_\nu \to 0 \Rightarrow I_\nu \to I_\nu(0)e^{-\tau_\nu} + \ S_\nu\tau_\nu$

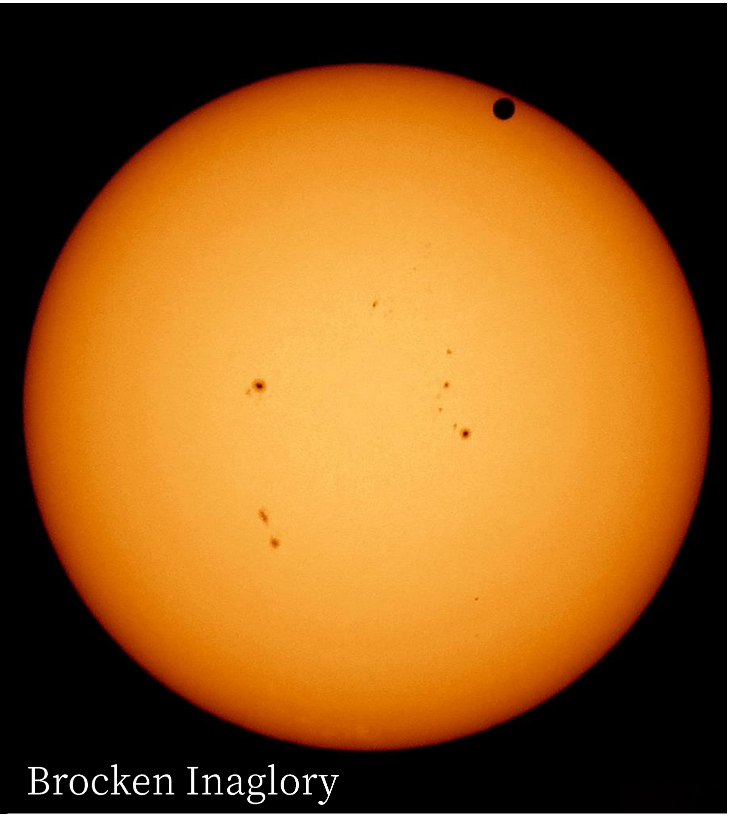
You can verify this by expanding $e^{-\tau_\nu}$ and take the first term.

Examples

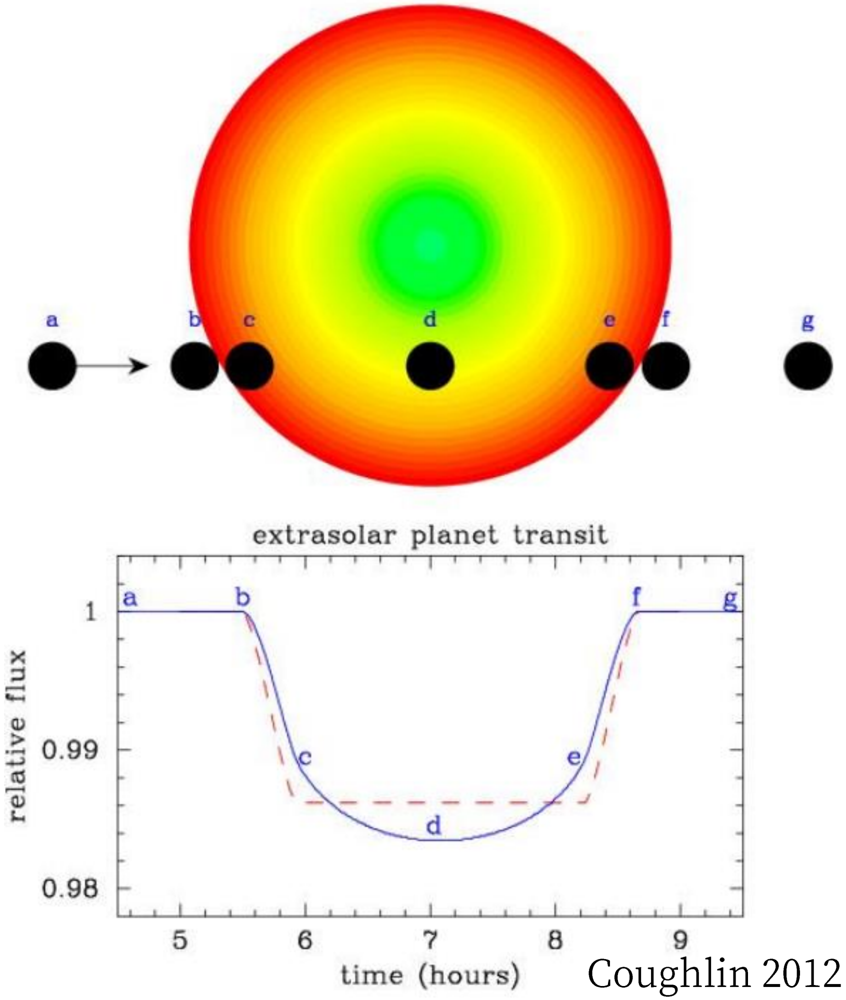
Limb darkening



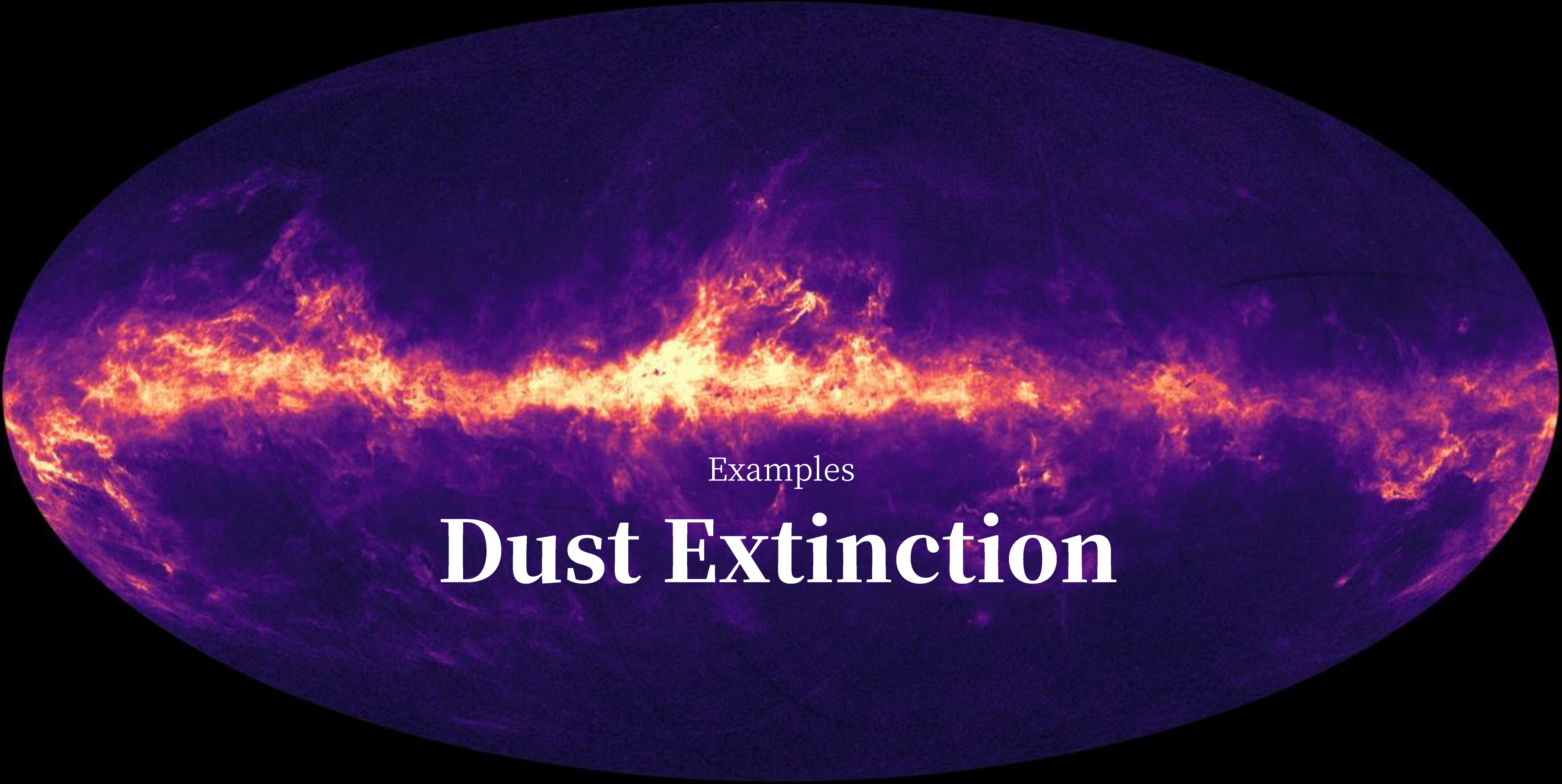
Credit: PAR



Brocken Inaglory



Coughlin 2012



Examples

Dust Extinction

Examples

Dust Extinction

Dust would extinguish photos.

This can be important in e.g. measuring distance.

The original distance modulus is:

$$m - M = 5 \log D - 5$$

But when there is dust, we should use

$$m - M = 5 \log D - 5 + A$$

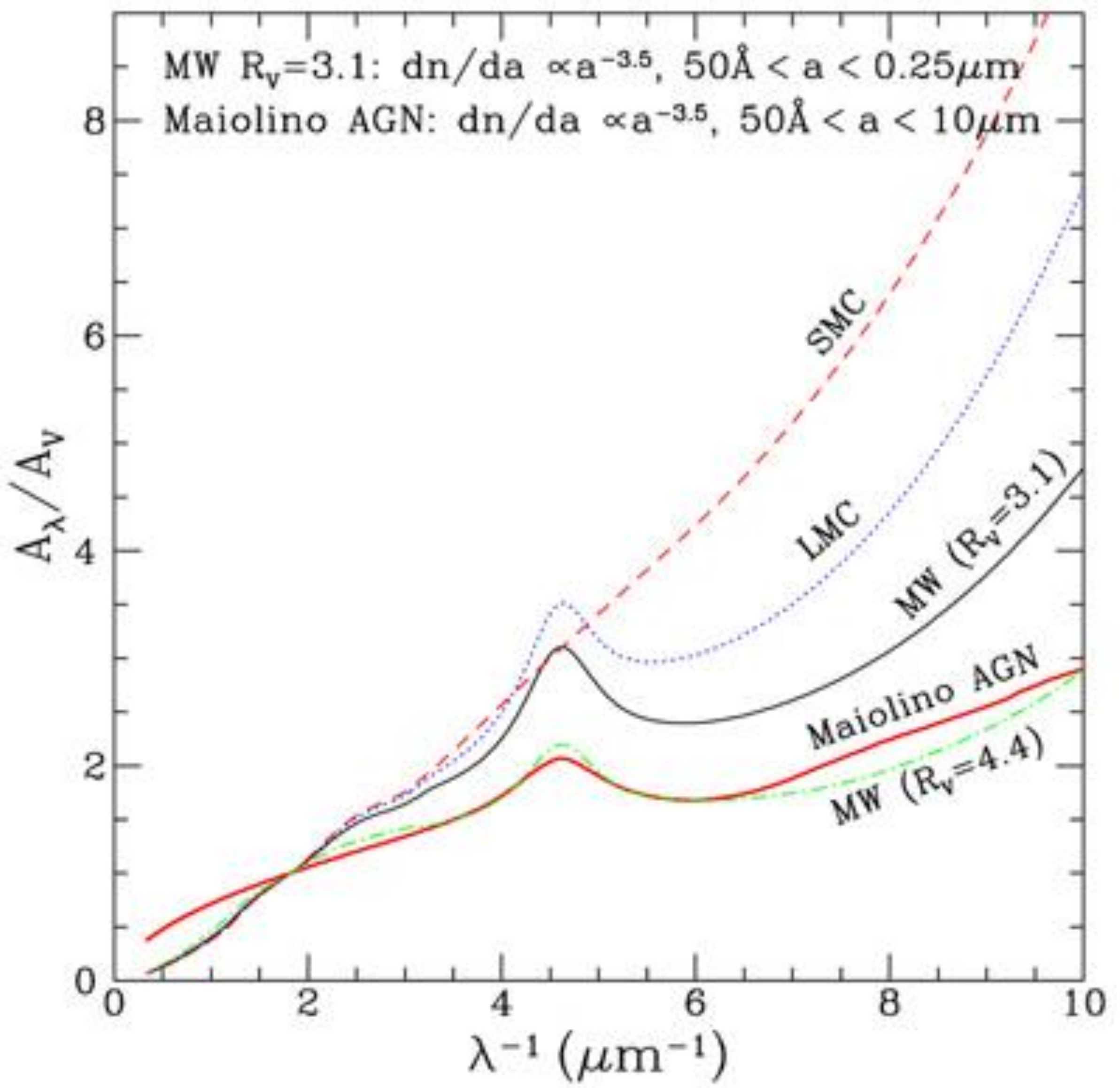
Examples

Dust Extinction

More importantly, extinction strength changes with wavelength.

In optical, the short wavelength light usually suffers stronger extinction, creating the **reddening effect**.

The wavelength dependence of extinction is called **extinction curve**.



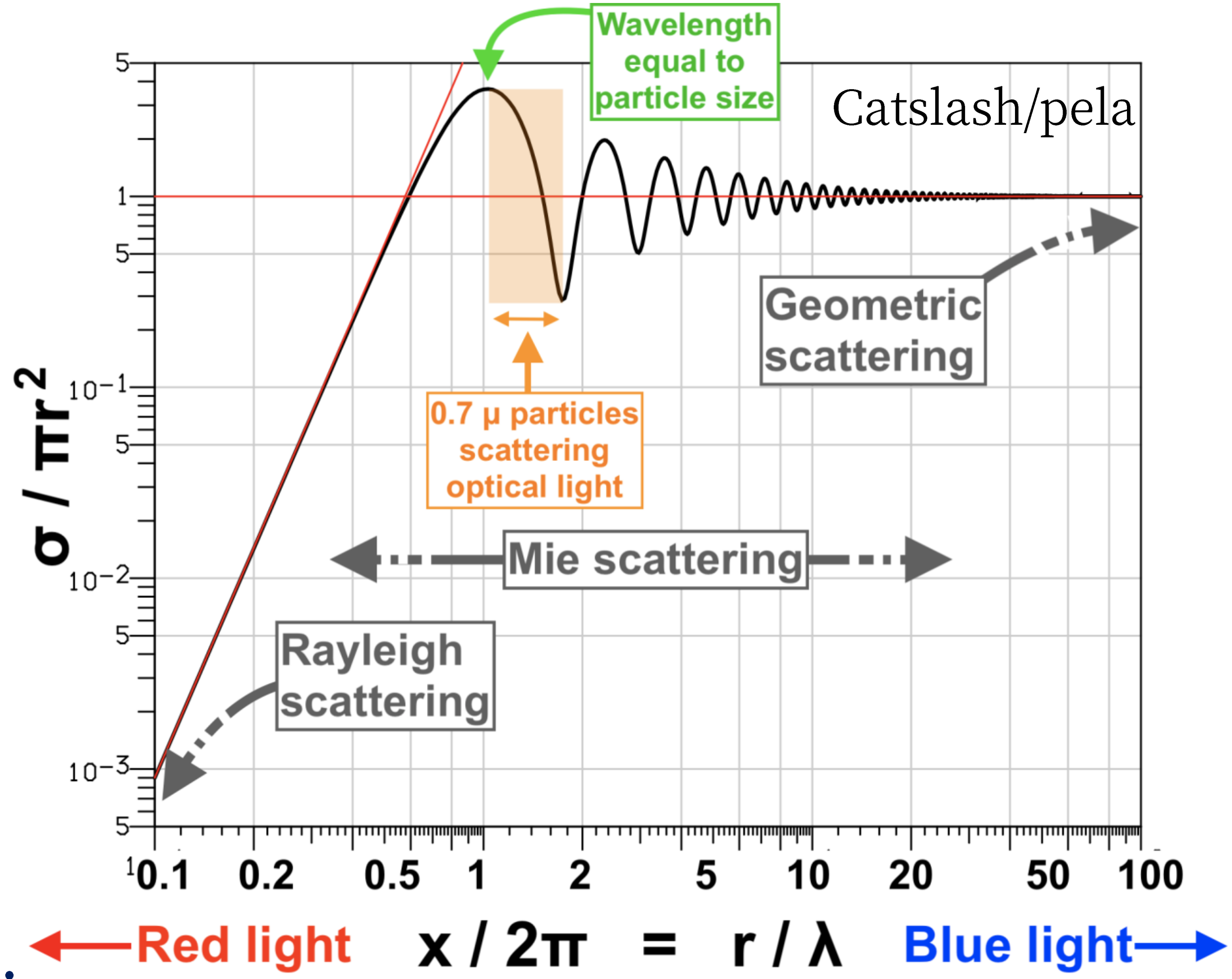
Nightmare

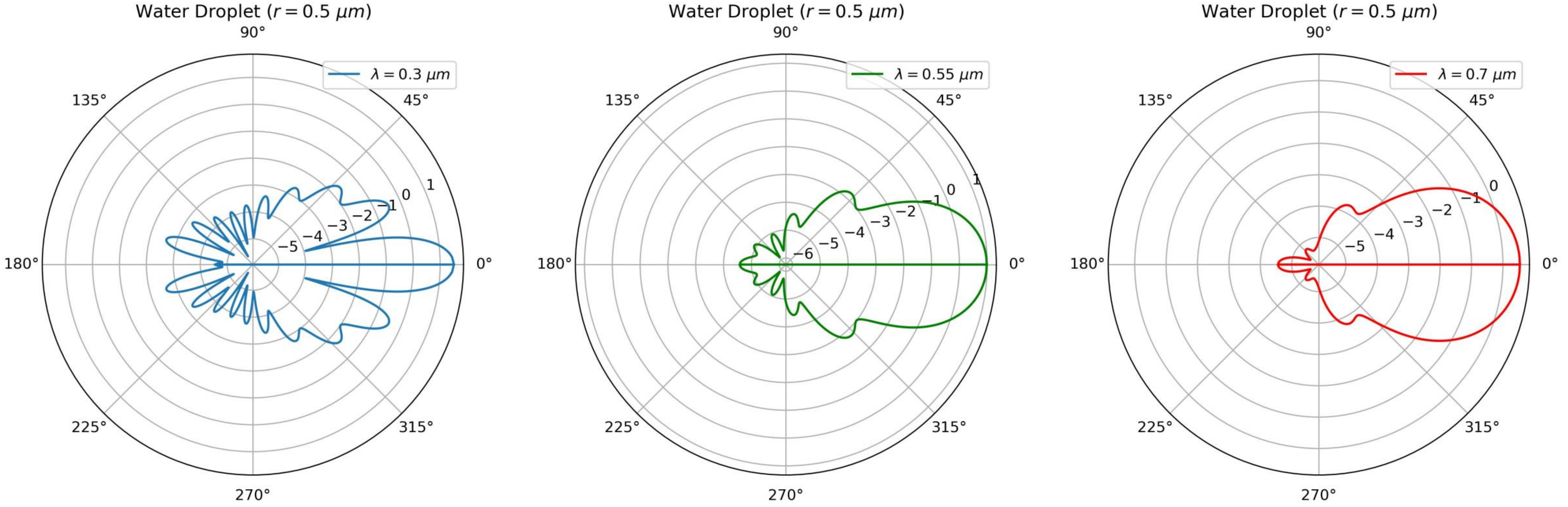
Scattering

In reality, photons can be scattered back into the line of sight, which fu*k up everything.

With scattering, our problem is no longer 1 dimensional.

Now, we have to consider the **complicated geometry** of our target.





Scattering is not only wavelength dependent, but also anisotropic.

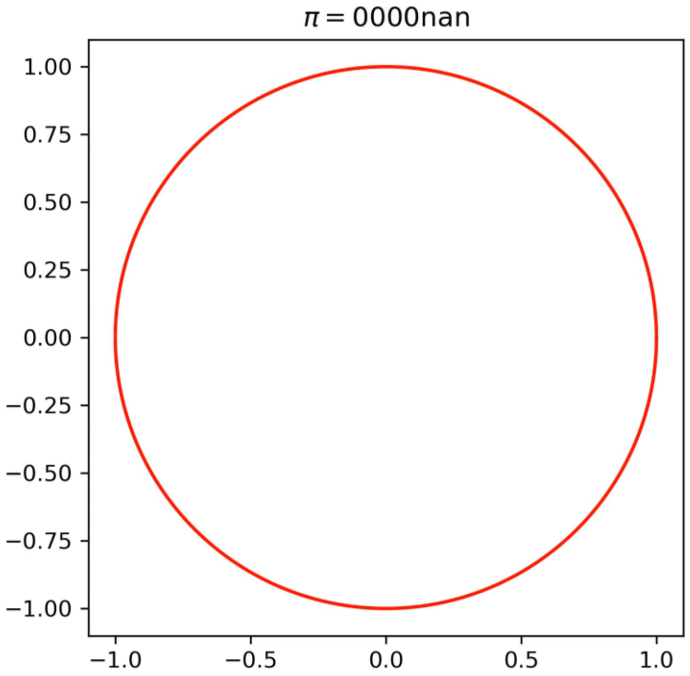
Different wavelength / grain size, creates different scattering pattern.

This is very hard to model analytically.

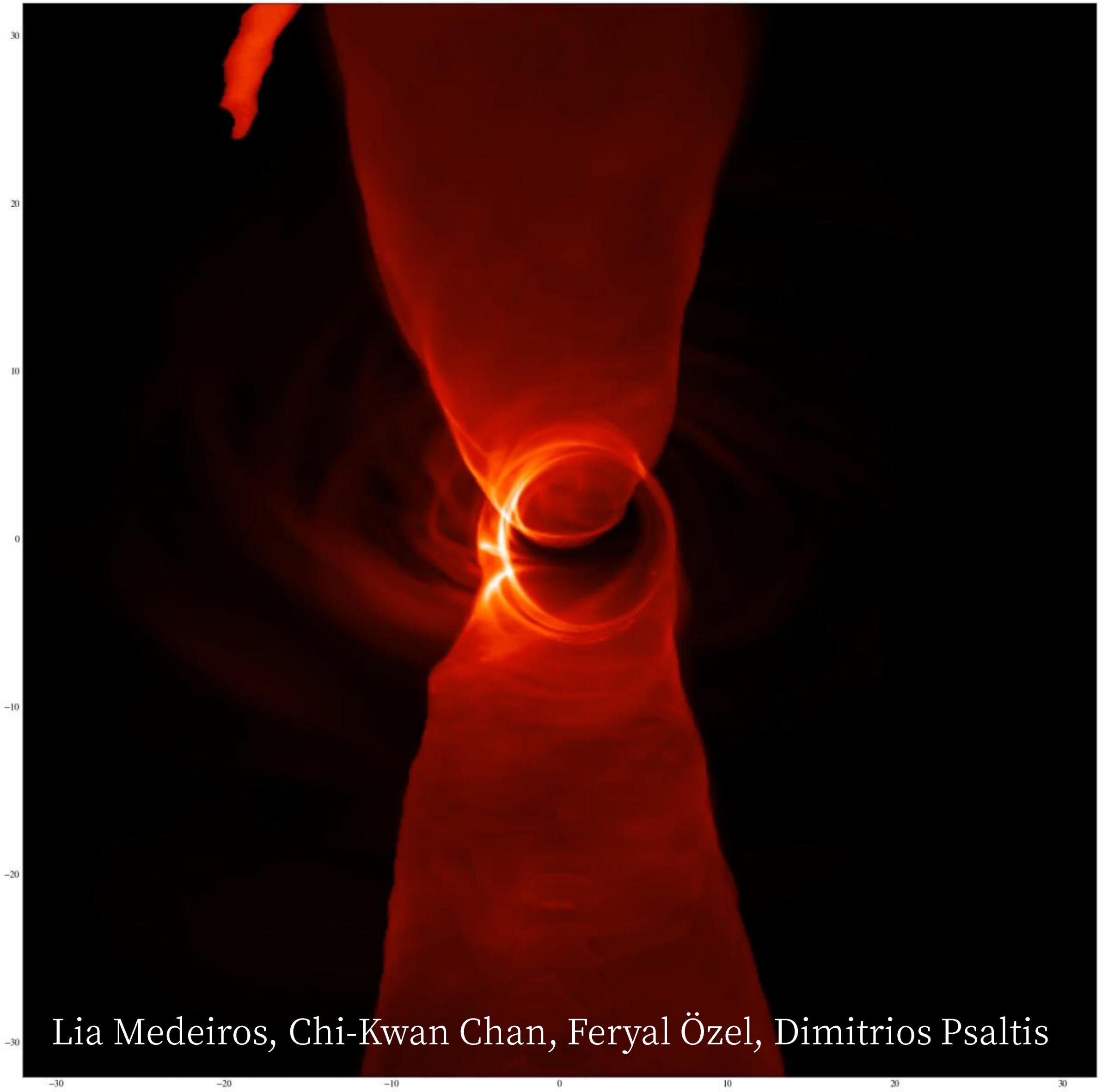
Computer go brrrrrrrrrr

Numerical Radiative transfer

Utilizing **Monte-Carlo method** and **Ray Tracing** to solve RTE.



Hotaka Shiokawa



Lia Medeiros, Chi-Kwan Chan, Feryal Özel, Dimitrios Psaltis

Summary

- In astrophysics, we often use **specific intensity** [erg cm^{-2} sr^{-1} s^{-1} Hz^{-1}] to describe the strength of light.
- Specific intensity is an **intrinsic property** of the source.
- Specific intensity is changed by **absorption**, **scattering** and **emission,** described by **radiative transfer equation**.
- Radiative transfer effects are often discussed using **optical depth**.
- Complicated radiative transfer problems can usually solved numerically.